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Module 6



Mathematics 6



Angles, Shapes, and Objects



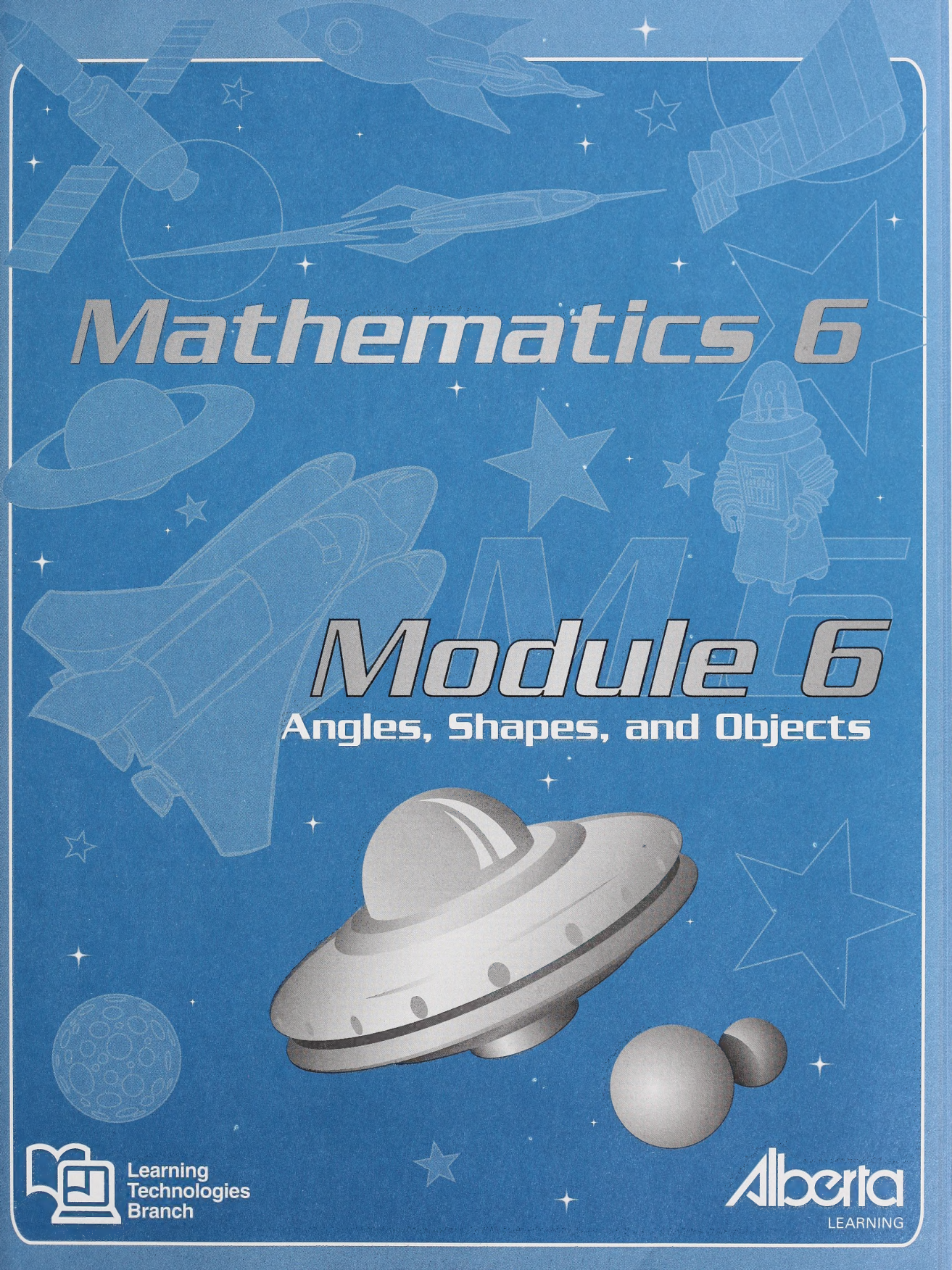
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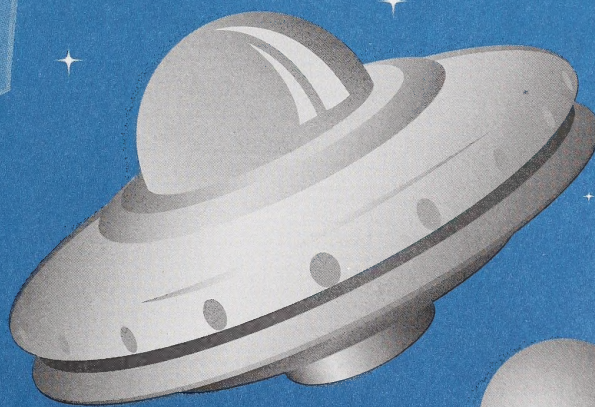
<https://archive.org/details/mathematics606albe>

The background is a deep blue space scene. It features faint line drawings of a satellite with solar panels, a rocket ship, a planet with rings, a space station, and an astronaut. Scattered throughout are white stars of various sizes. In the center, the numbers '1145' are faintly visible in a large, stylized font.

Mathematics 6

Module 6

Angles, Shapes, and Objects



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Mathematics 6
Module 6: Angles, Shapes, and Objects
Student Module Booklet
Learning Technologies Branch
ISBN 0-7741-2387-7

The Learning Technologies Branch acknowledges with appreciation the Alberta Distance Learning Centre and Pembina Hills Regional Division No. 7 for their review of this Student Module Booklet.

This document is intended for	
Students	✓
Teachers	✓
Administrators	
Home Instructors	
General Public	
Other	



You may find the following Internet sites useful:

- Alberta Learning, <http://www.learning.gov.ab.ca>
- Learning Technologies Branch, <http://www.learning.gov.ab.ca/lb>
- Learning Resources Centre, <http://www.lrc.learning.gov.ab.ca>

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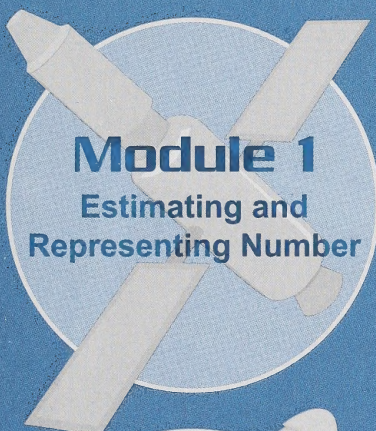
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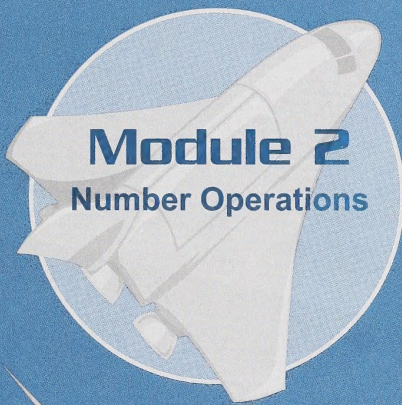
Welcome to **Mathematics 6**

Mathematics 6 contains nine modules.

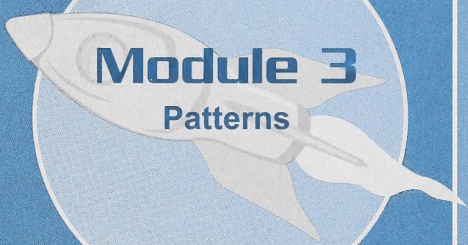
You should work through the modules in order (from 1 to 9) because concepts and skills introduced in one module will be reinforced, extended, and applied in later modules.



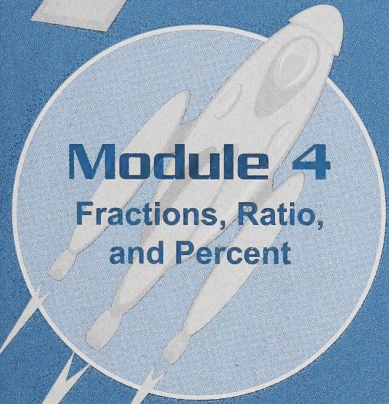
Module 1
Estimating and
Representing Number



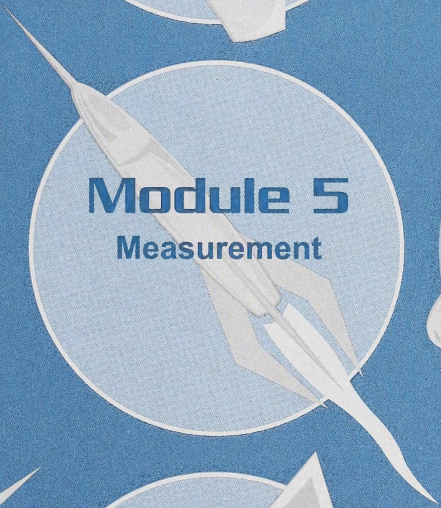
Module 2
Number Operations



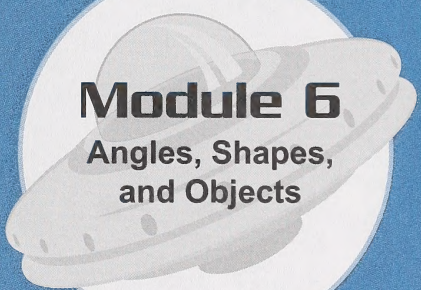
Module 3
Patterns



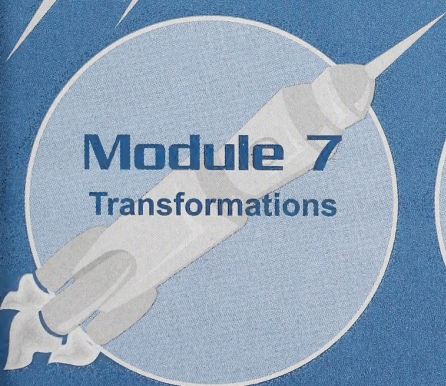
Module 4
Fractions, Ratio,
and Percent



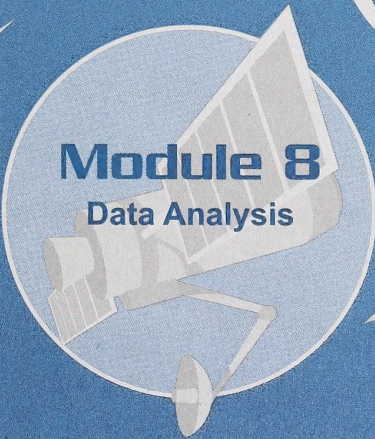
Module 5
Measurement



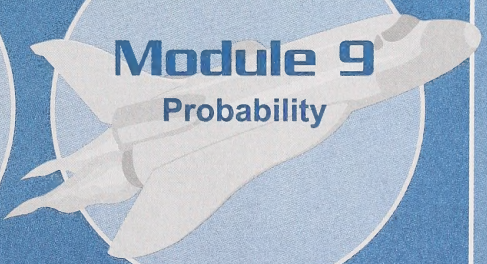
Module 6
Angles, Shapes,
and Objects



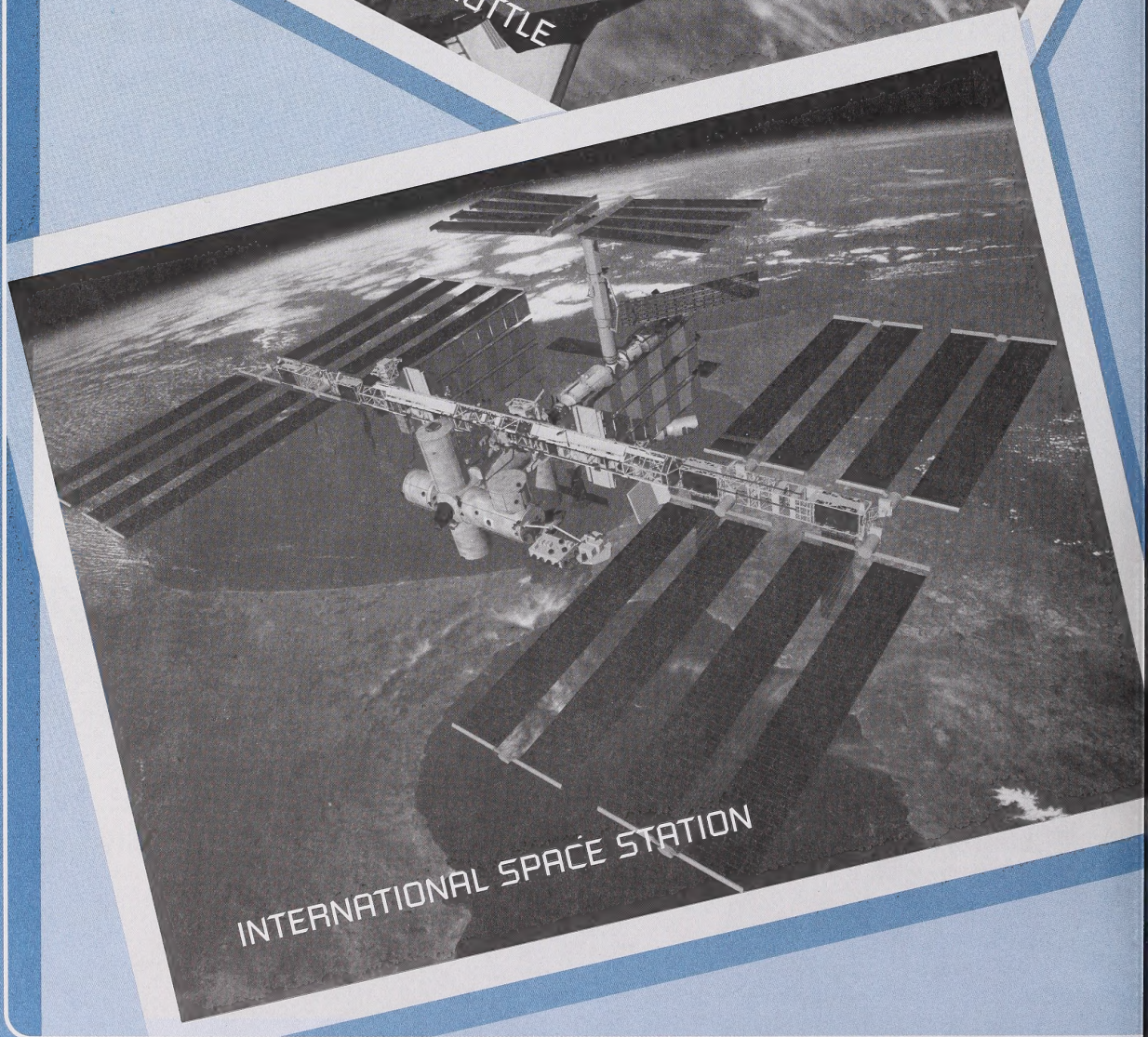
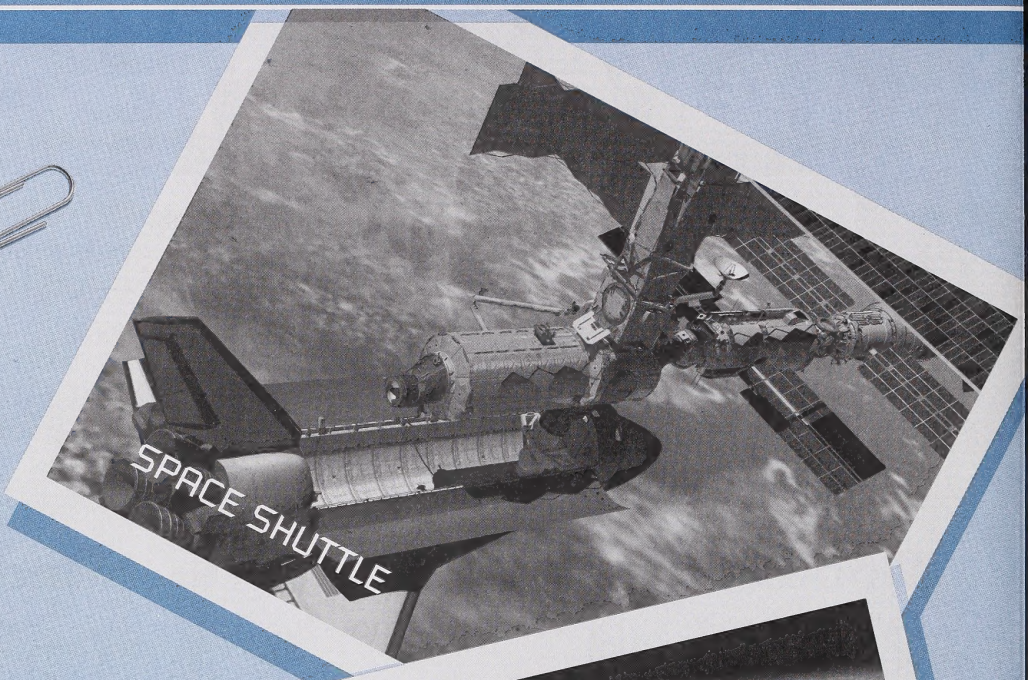
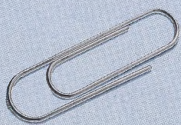
Module 7
Transformations



Module 8
Data Analysis



Module 9
Probability



Adventures in Outer Space

Matthew: Wow, what a wonderful experience it was meeting Colonel Chris Hadfield at the Odyssey! He gave a presentation here in Edmonton on July 9, 2001, and talked about his adventures in space, including his mission aboard the Space Shuttle *Endeavor* to attach Canadarm2 to the International Space Station.

It's too bad you missed it, Kylee. You were away visiting your grandmother in Slave Lake.

Kylee: My trip was great, but I sure wish I could have heard Colonel Hadfield talk about being the first Canadian to walk in space. But, I've got great news for you, Matthew! Commander Claire from the International Space Station is coming to town, and you and I will be spending some time with her.

I can't wait to hear about her adventures in space!

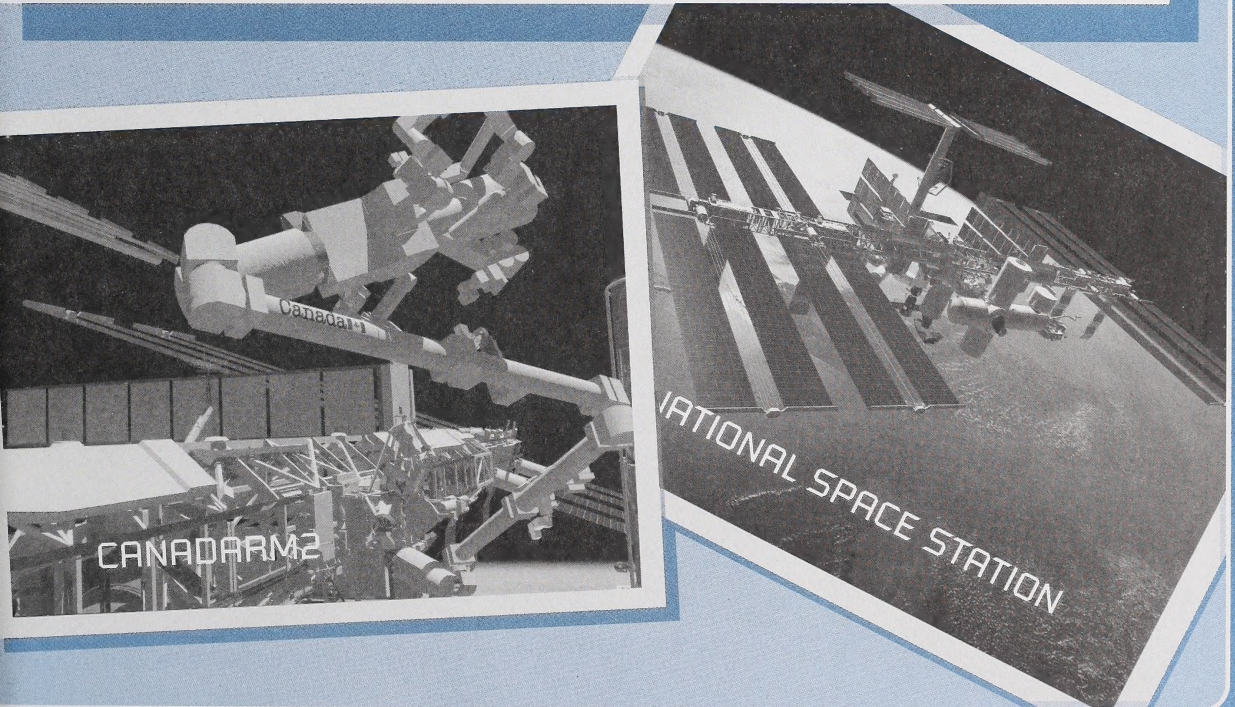


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Course Features



Take the time to look through the Student Module Booklets and the Assignment Booklets and notice the following design features:

- Each module has a Module Overview, Module Summary, and Review.
- Each module has several lessons. Each lesson focuses on a big idea that is central to the topic being learned in the module.
- Each lesson has several activities. An activity in each lesson is related to the Adventures in Outer Space theme.
- Each module has a Glossary and an Answer Key in the Appendix. In several modules there are also special pull-out pages in the Appendix.
- Each module has special exercises that focus on certain mathematical skills. The Numbers in the News project involves a scavenger hunt for samples of math in everyday life. The Keystrokes exercise introduces some “funky features” of the calculator that can be used to explore and practise important number ideas. Just the Facts gives you the opportunity to practise your basic number facts by doing a timed drill with your home instructor. The Mental Math exercise introduces an estimation skill or mental-computation strategy that you can use to sharpen your mental math skills.
- Each module references the Mathematics 6 Companion CD that includes additional material for review and mastery.

Required Resources

There are no spaces provided in the Student Module Booklets for your answers. This means you will need a binder and loose-leaf paper or a notebook to do your work.

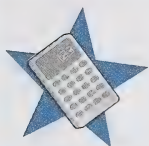
In order to complete the course, you will need a copy of the Mathematics 6 textbook, *Quest 2000: Exploring Mathematics, Grade 6*, the soft-cover book *Quest 2000: Exploring Mathematics: Practice and Homework Book, Grade 6*, a basic four-operation calculator (such as the TI-108 calculator), and various manipulatives (base ten blocks and pattern blocks).

If you wish to complete the optional computer activities, you must have access to a computer that is connected to the Internet.

You will also need access to a computer to view material on the Mathematics 6 Companion CD.

Visual Cues

For your convenience, the most important mathematical rules and definitions are highlighted. Icons are also used as visual cues. Each icon tells you to do something.



Use your calculator.



Use the Internet.



Refer to the textbook
or the Practice and
Homework Book.



Use the Mathematics 6
Companion CD.

Assessment and Feedback

The Mathematics 6 course is carefully designed to give you many opportunities to discover how well you are doing. In every activity you will be asked to turn to the Appendix to check your answers. Completing the activities and comparing your answers to the suggested answers in the Appendix will help you better understand math concepts, develop math skills, and improve your ability to communicate mathematically and solve problems.

If you are having difficulty with an activity, refer to the Answer Key in the Appendix for hints or help. As well as giving suggested answers to the questions, the Answer Key gives you more information about the questions.



Twice in each module you will be asked to give your teacher your completed assignments to mark. Your teacher will give you feedback on how you are doing.



After your teacher marks an assignment, be sure to review your teacher's comments and correct any errors you made.

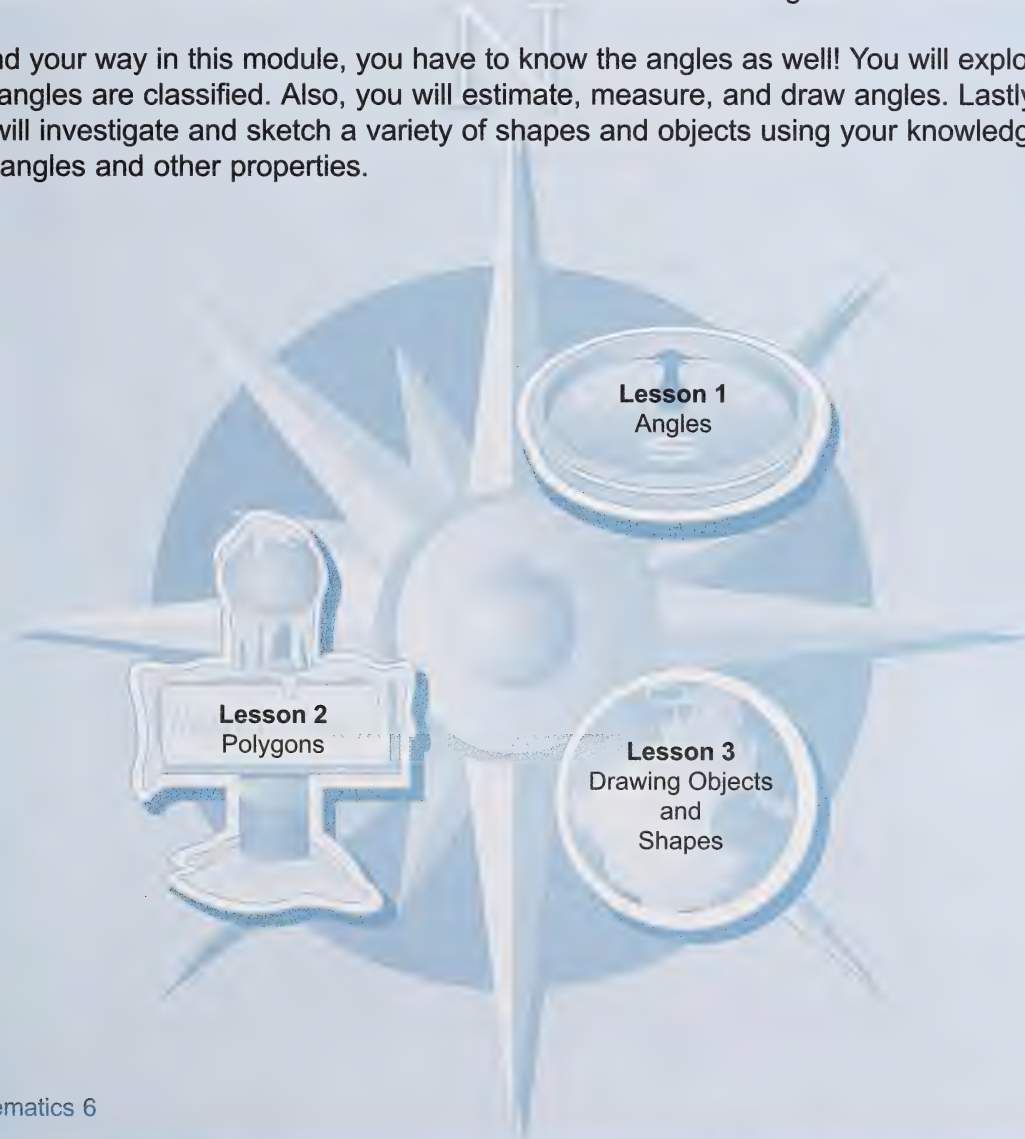
There will be a final test at the end of the course. You can prepare for the final test by completing the Review at the end of each module.

Module Overview

Tomorrow's space explorers will need navigational aids to travel safely into the unknown and return to Earth, just as the early explorers of Canada relied on their map-making skills and the magnetic compass to chart their journeys and return home.

Did you know that, in most parts of Canada, a magnetic compass does not point due north, but rather points to the North Magnetic Pole? For example, near St. John's, Newfoundland, the compass needle points at an angle of 23° towards the west, away from due north. Near Dawson, Yukon, it points at an angle of 27° towards the east. Travellers in the wilderness have to be able to add or subtract angles to find their way!

To find your way in this module, you have to know the angles as well! You will explore how angles are classified. Also, you will estimate, measure, and draw angles. Lastly, you will investigate and sketch a variety of shapes and objects using your knowledge of their angles and other properties.



Lesson 1 Angles

Lesson 2 Polygons

Lesson 3 Drawing Objects and Shapes

Your mark on this module will be determined by how well you complete the two Assignment Booklets.

The mark distribution is as follows:

Assignment Booklet 6A

Lesson 1 Assignment 30 marks

Lesson 2 Assignment 30 marks

Assignment Booklet 6B

Lesson 3 Assignment 30 marks

Numbers in the News 10 marks

Total 100 marks

When doing the assignments, work slowly and carefully. Be sure you attempt each part of the assignments. If you are having difficulty, you may use your course materials to help you, but you must do the assignments by yourself.

You will submit Assignment Booklet 6A to your teacher before you begin Lesson 3. You will submit Assignment Booklet 6B to your teacher at the end of this module.



Numbers in the News



Read through the following list before you begin Module 6. Begin by collecting samples of the ideas you already understand; others you may collect as you learn about them in the module. The samples you collect will depend on the newspapers or magazines you use.

Scavenger Hunt

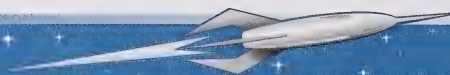


Cut out articles or advertisements from newspapers or magazines that show measurements being used in different situations. Here are some suggestions of things to look for:

- pictures of angles of different sizes (acute, right, obtuse, straight, and reflex angles)
- pictures of different kinds of triangles being used or shown in a variety of ways
 - equilateral, isosceles, and scalene
 - acute, right, and obtuse
- pictures of different kinds of quadrilaterals
 - trapezoids, parallelograms, rectangles, rhombuses, kites, and squares
- pictures of different kinds of polygons
 - regular and irregular
 - pentagons, hexagons, heptagons, octagons, and others
 - with or without lines of symmetry
- pictures of 3-D objects, such as prisms and pyramids

You will find further instructions for completing and submitting your project in Assignment Booklet 6B.

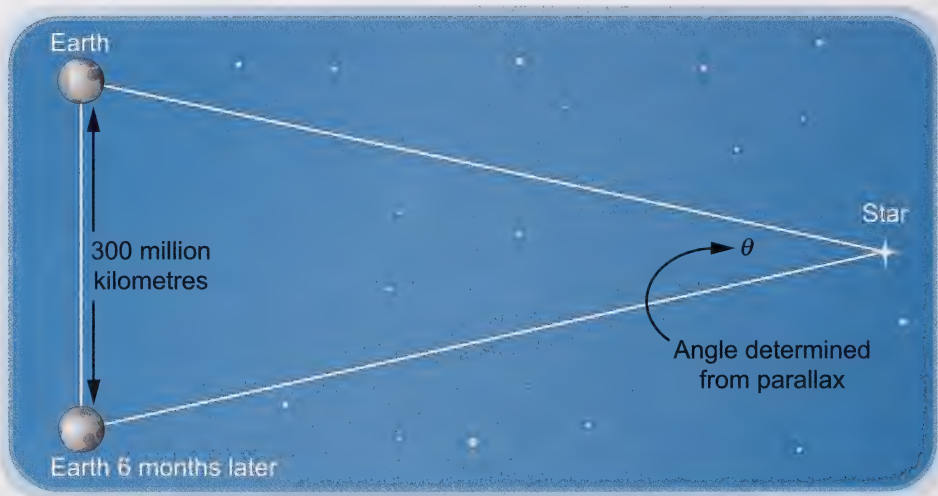
Lesson 1



Angles

Have you ever wondered how astronomers know how far the stars are from Earth? For the stars closest to Earth, scientists use a principle called **parallax**.

Stretch your arm straight out in front of you. Hold up one finger. Close one eye and line up your finger with a distant object, such as a lamppost. Now close that eye and open your other eye. Your finger appears to shift away from the object you had originally lined it up with. This apparent shift is called parallax.



Astronomers carefully observe the apparent positions of the closest stars every six months. In half of a year, Earth travels 300 million kilometres. This distance provides astronomers with the length of one side of a very large triangle. The apparent shift of a star from its location between observations is used to determine the size of the very small angle in the triangle located at the star. Then astronomers use a mathematical calculation to determine the star's distance.

In this lesson you will learn how to classify angles, estimate and measure angles, and draw angles. You will identify many different kinds of angles that are used in your environment.

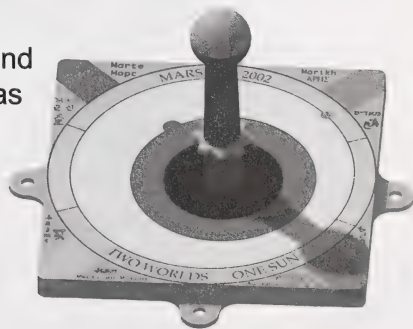
Activity 1

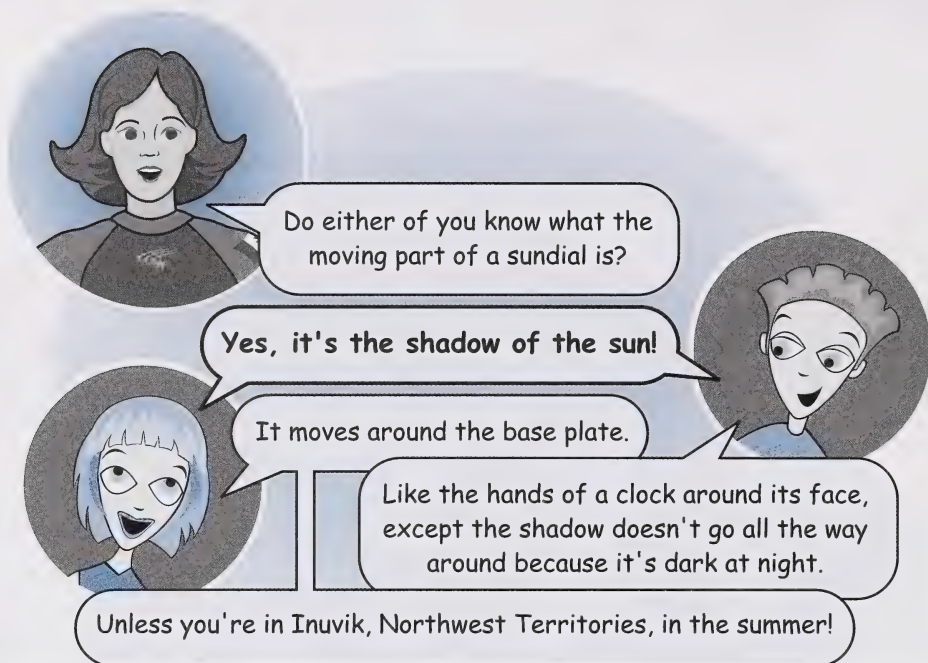


Today you will classify angles.



Commander Claire was talking with Kylee and Matthew. “You kids are right—sundials are as old as the hills, but you may be surprised to know that we have one just like this on Mars! We perform lots of math calculations to make sure special adjustments are done to make the sundial work properly.”





If you have access to the Internet, you can find out more about the Martian sundial at the following website:

<http://www.washington.edu/newsroom/news/1999archive/04-99archive/k042199.html>

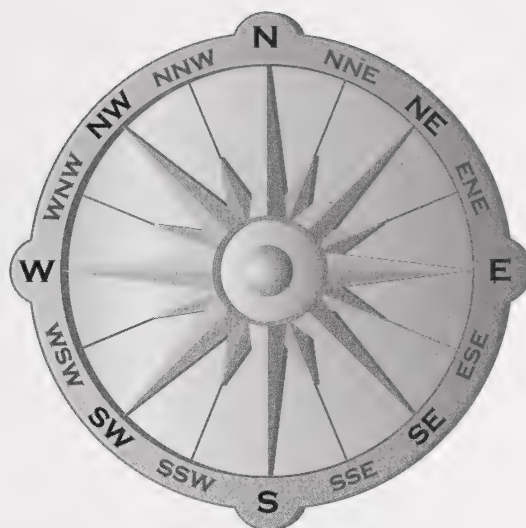
For more information about how sundials work, visit the following website:

<http://liftoff.msfc.nasa.gov/Academy/Earth/Sundial/Sundial-how.html>



Turn to page 204 in your textbook and look at the sundial shown there. The piece centred on top of the sundial is called a **gnomen**. Notice that you can tell the time by looking at the shadow of the gnomon. As Earth rotates, the length and angle of the shadow changes. Turn to page 207 in your textbook and look at the picture and read the definition of an angle in the box at the bottom of the page.

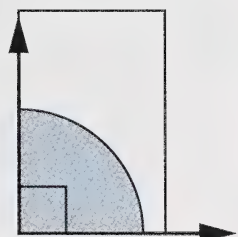
This picture shows a sundial that is similar to the one on page 204 of your textbook. Sixteen directions are marked with rays. These rays share a vertex at the centre of the sundial and form many angles.



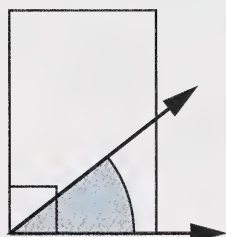
Cut a rectangular sheet of tissue paper or tracing paper in half and mark the square (90°) corners as shown.



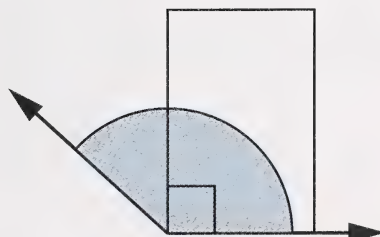
You can use these corners to decide if an angle is equal to 90° , less than 90° , more than 90° , equal to 180° , or more than 180° .



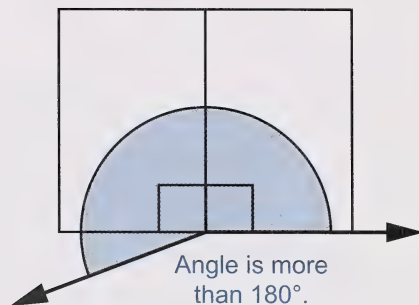
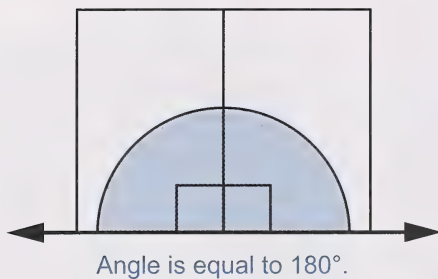
Angle is equal to 90° .



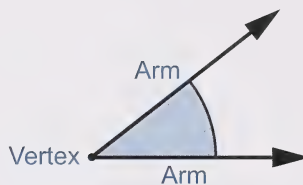
Angle is less than 90° .



Angle is more than 90° ,
but less than 180° .

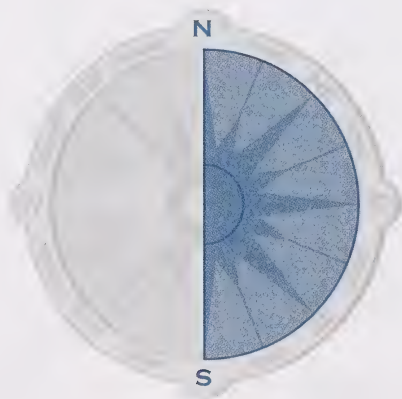


The two rays of an angle are called its **sides** or **arms**. The common endpoint of the two rays is called the **vertex** of the angle.

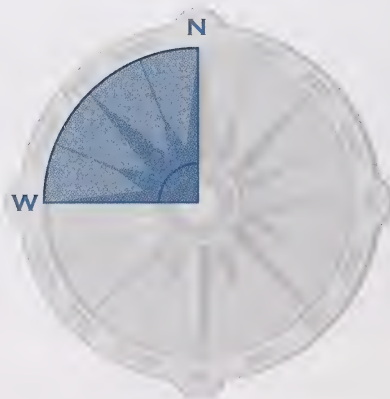


1. Use your square corners to decide if each of the following angles is equal to 90° , less than 90° , more than 90° but less than 180° , equal to 180° , or more than 180° .

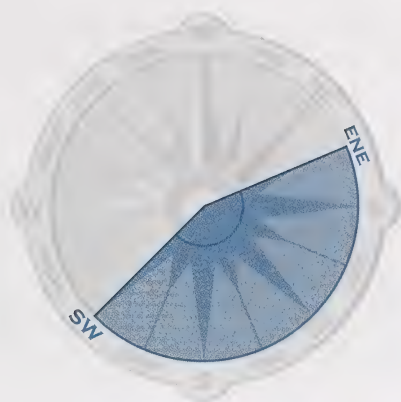
a.



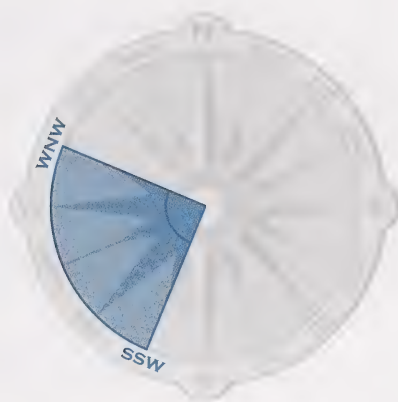
b.



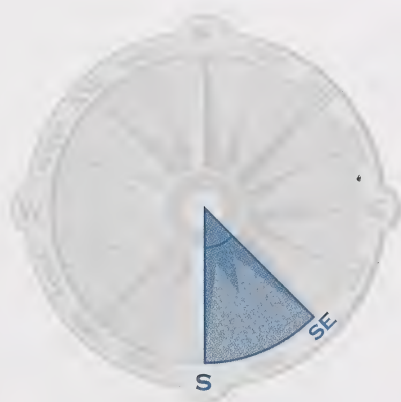
c.



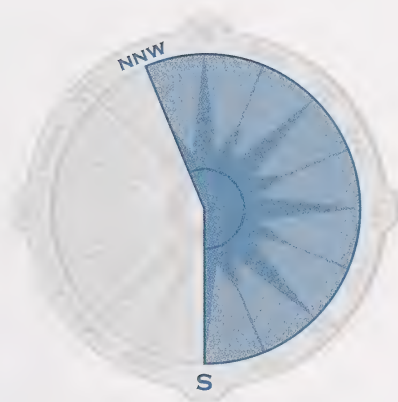
d.



e.



f.



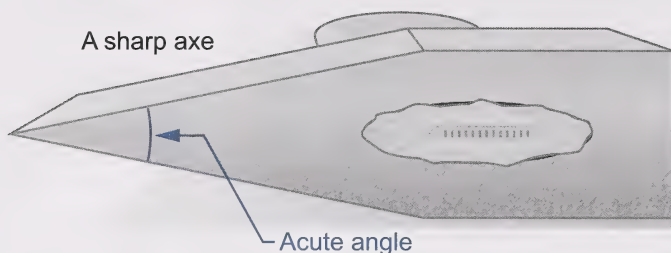
2. Turn to page 209 in your textbook and look at the pictures and read the definitions of different kinds of angles. Classify each of the angles in question 1 as **right**, **acute**, **obtuse**, **reflex**, or **straight**.

Check your answers on page 77 in the Appendix.

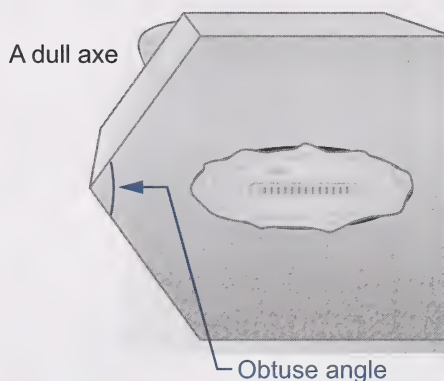
Look at the room in which you are sitting. Do you see any right angles? If the carpenters were careful, the angle where two walls meet will be a **right angle**, or 90° in measure.

Do you agree that if you turn your chair through two right angles, you will be facing in the opposite direction? A **straight angle** consists of two right angles and is 180° in size.

If you look at the head of an axe from above, the sides of the blade form an **acute angle**. The word *acute* means “sharp.” An acute angle is a sharp angle that is less than 90° in measure.



An **obtuse angle** is an angle that is between 90° and 180° . The word *obtuse* means “dull.” How sharp do you think an axe would be if the sides of the blade formed an obtuse angle?



3. For each of the sundials in question 1, draw an arc that shows a different angle formed by the same two rays (by going around the other way), and classify it as **right**, **acute**, **obtuse**, **reflex**, or **straight**.
4. Explain why two different angles can be formed by any two rays.
5. Classify (as **right**, **acute**, **obtuse**, **reflex**, or **straight**) the other angle formed by a pair of rays if one angle is
 - a. an acute angle
 - b. a straight angle
 - c. an obtuse angle
 - d. a right angle
 - e. a reflex angle

Check your answers on pages 77 and 78 in the Appendix.

6. Name all the possible rays that could form each of the following angles on a sundial.

- a. an obtuse angle with the NE ray
- b. a reflex angle with the SSW ray
- c. a straight angle with the ENE ray
- d. an acute angle with the W ray
- e. a right angle with the SW ray



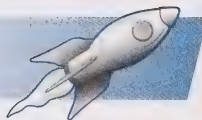
7. Turn to page 214 in your textbook and look at the clock. Tell what time it is and classify the angle formed by the hands.



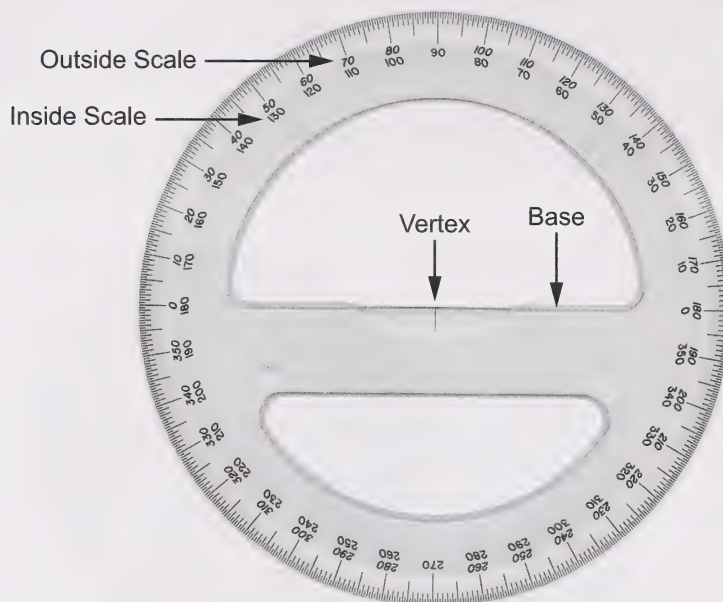
8. Classify the two angles formed by a clock's hands (going around both ways) when it reads the following times.
- a. 4:30
 - b. 8:15
 - c. 9:00
9. With the minute hand on 12 and the hour hand on an hour, give a time when an angle formed by the clock hands is
- a. right
 - b. acute
 - c. obtuse

Check your answers on page 79 in the Appendix.

Activity 2



Today you will estimate the sizes of angles and then use a protractor to determine the actual sizes of the angles.

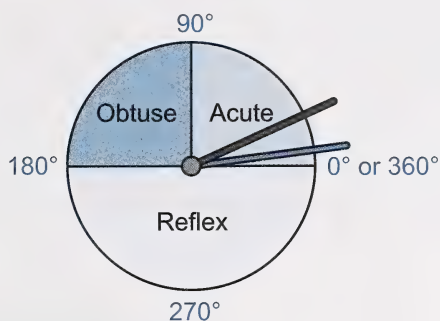


In the last activity you classified angles as being right, acute, obtuse, reflex, or straight. Now you will explore angles by building a simple tool.

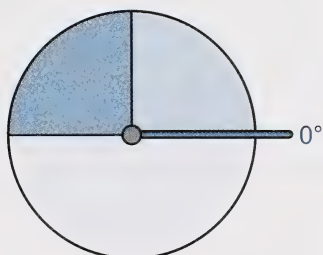
- Step 1:** Use a circular object with a diameter of about 10 cm to trace a circle on a sheet of paper.
- Step 2:** Cut out the circle and fold it in fourths (by folding it in half twice).
- Step 3:** Unfold the paper circle and use a ruler to draw over the fold lines.
- Step 4:** Paste the paper circle on a piece of light cardboard (from a cereal box). Cut around the edge of the circle.

Step 5: Cut out two thin strips of paper. Fasten one end of each strip to the centre of the circle with a tack so that you can rotate the strips around the circle.

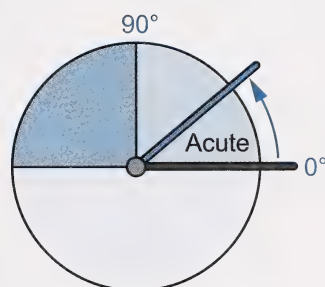
Step 6: Colour and label the circle as shown.



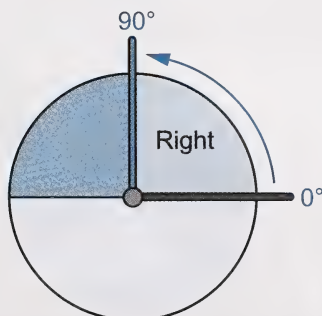
Now you have a tool that can help you determine types of angles and estimate their measures.



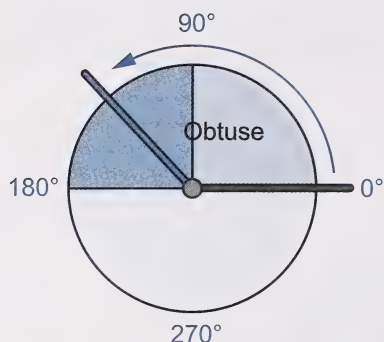
You can think of overlapping rays as being an angle of 0° .



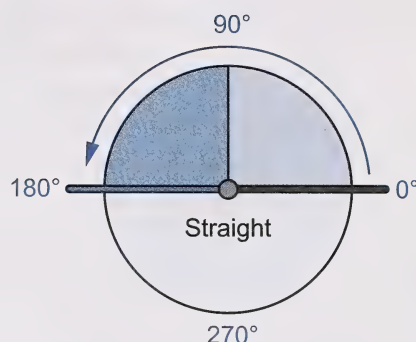
You could estimate this angle to be about 40° because it is a little less than halfway between 0° and 90° .



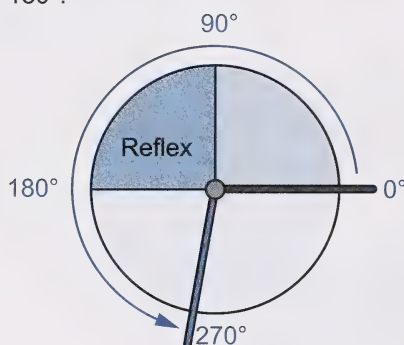
This angle is 90° because its rays are perpendicular to each other.



You could estimate this angle to be about 135° because it opens up about halfway between 90° and 180° .

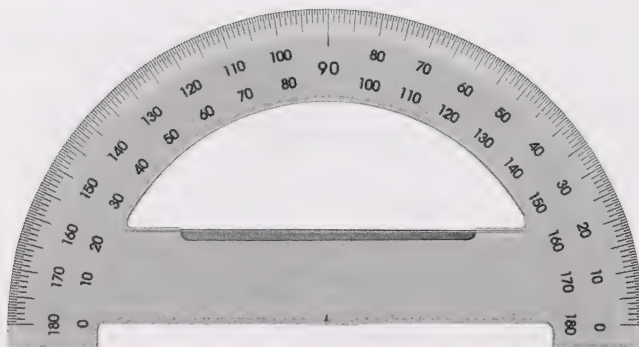


This angle is 180° because its rays form a straight line.

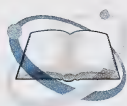


You could estimate this angle to be about 260° because it opens up a little less than 270° .

To measure angles more accurately, it would help if the circle were divided into many more smaller sections. In fact, this is what a full-size **protractor** is—a circle with number scales showing all 360 divisions of a circle, which are called degrees. However, most protractors are half circles and only show 180° .



All protractors have two scales running in opposite directions. One scale goes clockwise and the other goes counterclockwise. This allows you to measure angles that open in different directions. You must be careful to use the appropriate scale. It is wise to estimate the size of the angle first so that you know which protractor scale to read.



Turn to page 213 in your textbook to Using a Protractor. The picture shows you how to use a protractor to measure an angle. Place the protractor on top of the angle. Position it so that the vertex marking on the protractor lies on the vertex of the angle, and the 0° ray of the protractor lies on one ray of the angle. Follow around the edge of the protractor from 0° , perhaps counting up by 10s, until you reach the other ray of the angle. Then read the angle on the protractor.

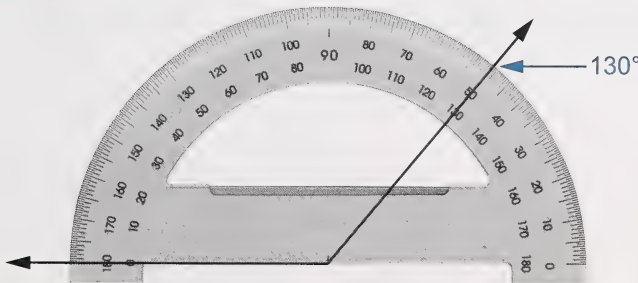
Consider the following examples.

Example

Use a protractor to measure the following angle.



Centre the protractor on the vertex of the angle, and align the base of the protractor with one of the arms of the angle as shown. Because the angle is obtuse, it will be greater than 90° in measure. Use the inner scale to measure the angle. The angle is 130° .

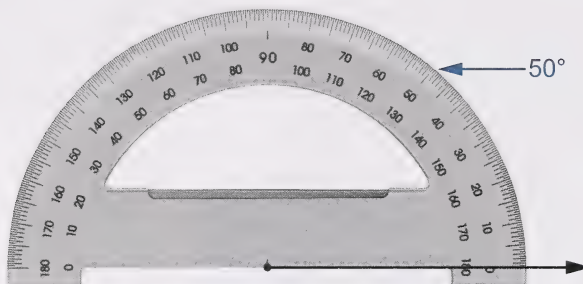


Example

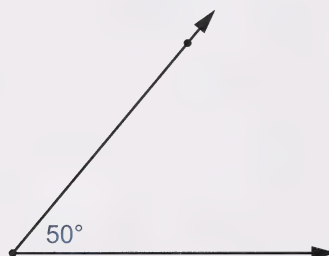
Use a protractor to draw a 50° angle using the given ray as the lower arm of the angle.



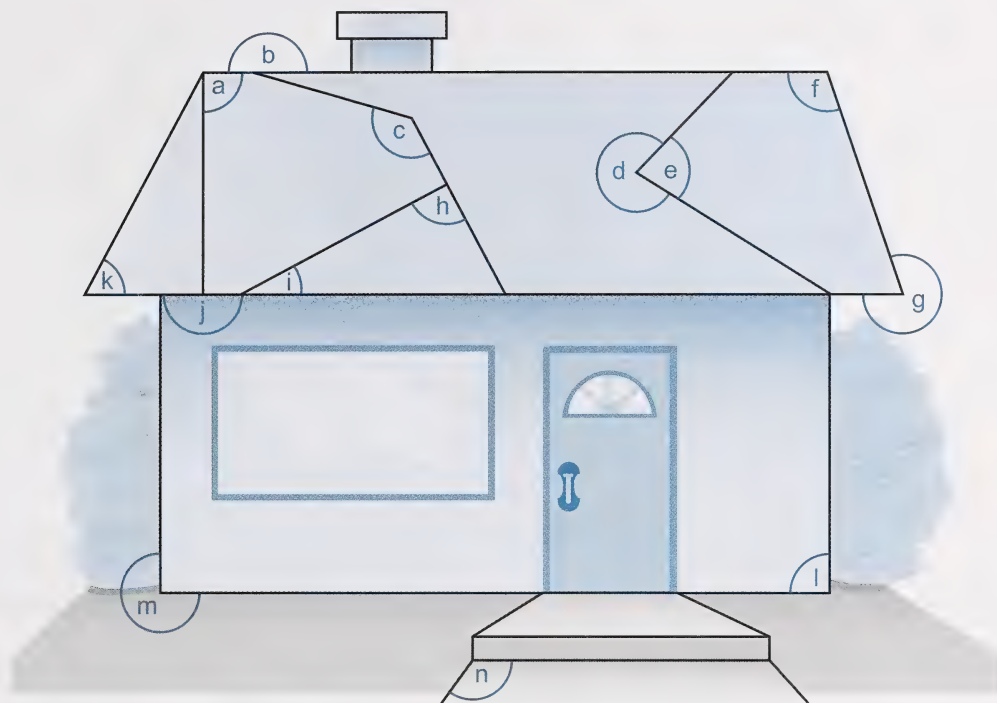
Centre the vertex of the protractor on the end of the ray. Align the base of the protractor with the ray as shown.



Because a 50° angle is acute, you will count upward from 0° using the outside scale. Put a dot at 50° and draw the second arm.



1. Estimate and then measure each of the labelled angles in the following diagram. (**Hint:** You may need to extend short rays so they reach the scale when the protractor is placed on top.)



2. Use the picture in question 1. Find and label angles that have each of the following measures.

a. angle o = 62°

b. angle p = 32°

c. angle q = 16°

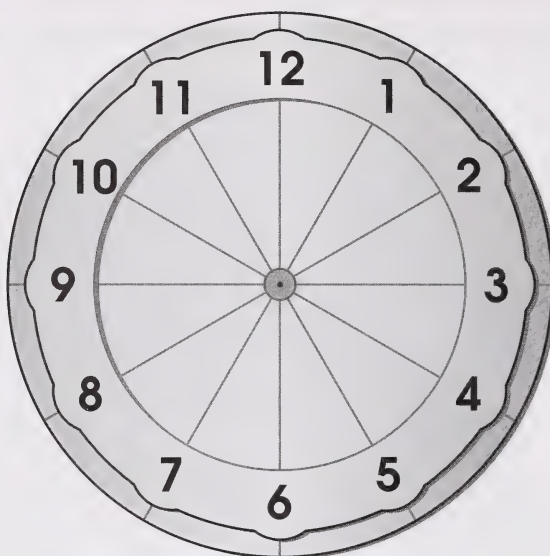
d. angle r = 118°

e. angle s = 148°

f. angle t = 152°

Check your answers on page 79 in the Appendix.

Use the following picture of a clock face to answer questions 3 to 10.



3. How many degrees does the minute hand move in 1 h?
4.
 - a. Use your protractor to measure how many degrees the hour hand moves in 1 h.
 - b. Use your answer to question 3 to explain how to get the correct answer to question 4.a. without using a protractor.
 - c. How many minutes does it take the minute hand to move the same number of degrees as the hour hand moves in 1 h?
5.
 - a. Use your protractor to measure the number of degrees the minute hand moves in 20 min.
 - b. Explain how to get the correct answer to question 5.a. without using a protractor.
6. Explain how you can find how many degrees the minute hand moves in 1 min without using a protractor.

Check your answers on page 80 in the Appendix.

- 7. a.** Use your protractor to measure a 45° angle on the face of the clock. Estimate from the picture how many minutes it would take for the minute hand to move 45° .
- b.** Explain how to get the exact number of minutes for question 7.a. without using a protractor.

Questions 8 to 10 are optional. Do these questions if you need more practice.

- 8. a.** Measure how many degrees the minute hand moves between 5 min after an hour and 40 min after.
- b.** Explain how to get the exact number of degrees for question 8.a. without using a protractor.
- 9.** If the minute hand is on the 12, at what times would a 60° angle be formed?
- 10. a.** Estimate how many degrees the hour hand moves through in 20 min.
- b.** Explain how to get the exact number of degrees for question 10.a. without using a protractor.

Check your answers on pages 80 and 81 in the Appendix.

Sharing Time

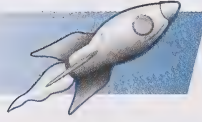
Now it's time to show your home instructor what you have been learning.



Turn to page 93 of the Practice and Homework Book and complete questions 1 to 4.

Discuss your answers with your home instructor.

Activity 3

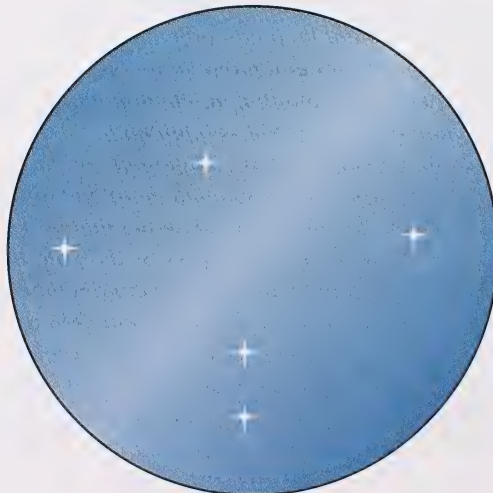


Today you will explore angle measurement in more detail.



Commander Claire enjoys gazing at the stars through her telescope. She imagines drawing rays using the stars as endpoints.

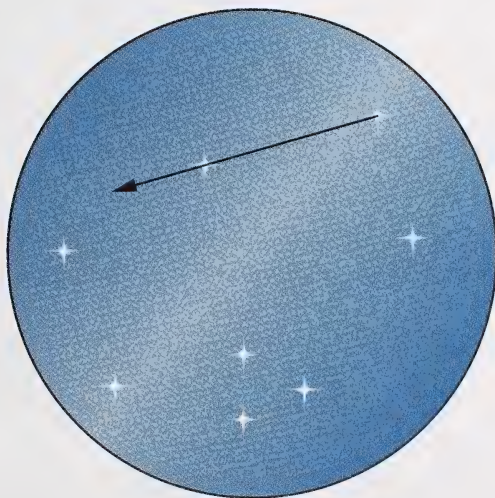
1. The picture below shows the stars Commander Claire first saw through her telescope one evening. She noticed that two rays with the same endpoint formed a right angle. Use estimation to find and draw the right angle, and then measure it to verify your estimate.



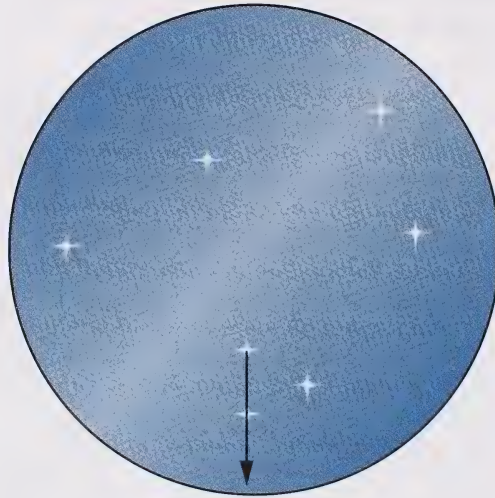
2. When the sky darkened and more stars appeared, Claire noticed that two rays with the same endpoint formed a straight angle. Use estimation to find and draw the straight angle, and then measure it to verify your estimate.



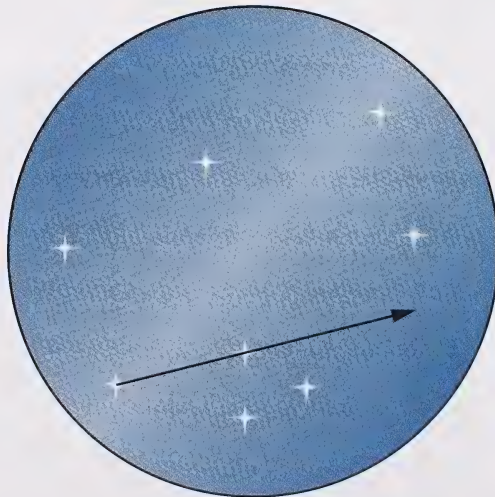
3. The following picture shows one ray that Claire visualized. Use estimation to find and draw another ray with the same endpoint so that the two rays form a 45° angle. Measure it to verify your estimate.



4. Use estimation to find and draw another ray with the same endpoint so that the two rays form a 120° angle. Measure it to verify your estimate.

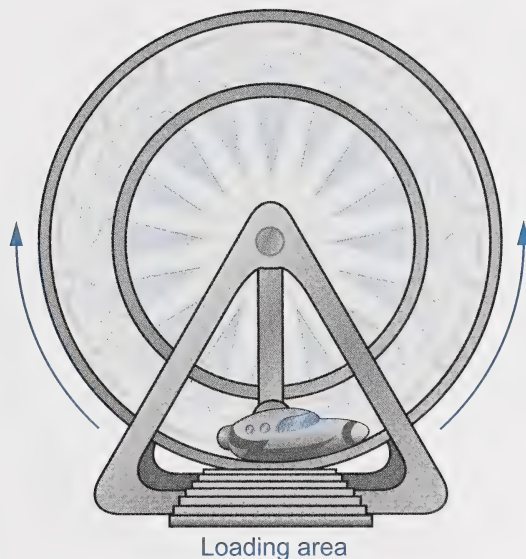


5. Use a protractor to draw another ray with the same endpoint so that the two rays form a 60° angle. Draw a dot to show where a star could be on the ray you drew.



Check your answers on page 81 in the Appendix.

Commander Claire and her crew spent months riding in space simulators to get used to being turned upside down and spun around. One simulator resembles a ferris wheel. It loads from the bottom and can turn in either direction.



For questions 6 to 8, draw a blue line to show the angle at which Claire is tilted after turning the number of degrees given from the starting position.

6. 35° counterclockwise

7. 115° clockwise



8. 290° clockwise



Loading area

For questions 9 to 11, draw a red line in the pictures for questions 6 to 8 to show the angle at which Claire is tilted after making the turns described.

9. From her end position in question 6, Claire turned 145° counterclockwise.

10. From her end position in question 7, Claire turned 90° counterclockwise.

Questions 11 and 12 are optional. Do them if you need more practice.

11. From her end position in question 8, Claire turned 350° clockwise.

12. What is the least number of degrees Claire must turn to get to the loading position

- a. after question 9?
- b. after question 10?
- c. after question 11?

Check your answers on pages 82 and 83 in the Appendix.

Challenge Activity



What is the first time between 2:00 and 3:00 that the hands on a clock form a 160° angle?



Check your answers on pages 83 to 85 in the Appendix.

Conclusion

In this lesson you learned how to classify angles, estimate and measure angles, and draw angles. You saw that many different kinds of angles are used in the world around you. One use of angles is to measure the distance from Earth to the nearby stars. Did you know that the first accurate measurement of the distance to a star occurred in 1838, less than 200 years ago?

Turn to Assignment Booklet 6A and complete the Lesson 1 Assignment.

Keep Assignment Booklet 6A until you have completed the entire booklet.

Polygons



City Hall - Edmonton, Alberta

If you look at modern architecture, you will see more than the simple rectangular designs that often make buildings look like cardboard boxes. The photograph above shows Edmonton City Hall. A variety of geometric forms makes the building appealing to look at—a place of interest rather than just a place to work or do business. In the glass and steel pyramid that forms the main structure, can you pick out triangles, quadrilaterals, and other polygons? If you choose architecture as a career, you will have to learn about many different shapes and their properties.

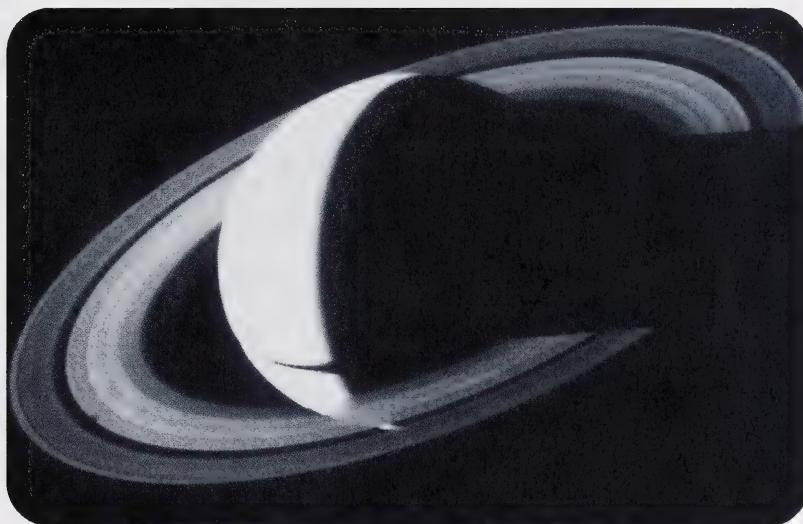
In this lesson you will extend what you know about triangles, quadrilaterals, and other polygons. You will classify triangles according to the sizes of their angles. You will investigate symmetry in polygons and use this property to classify quadrilaterals.

Activity 1



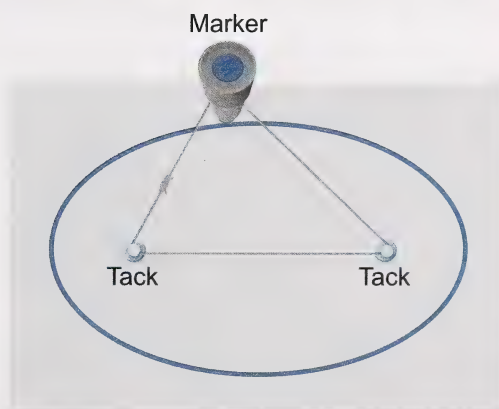
Today you will classify triangles by the lengths of their sides and the measures of their angles.

Did you know that triangles are related to the orbits of the planets around the sun and the shape of Saturn's rings?



NASA/JPL

Take a sheet of cardboard and push two tacks into the board about 20 cm apart. Get a piece of string that is about 50 cm long and tie its ends together to form a loop. Place the loop around the tacks and stretch it out using a coloured marker to form a triangle. Now use the marker to draw a curve while keeping the string taut.



The egg-shaped curve you draw is called an **ellipse**. The positions of the tacks are called the **foci** of the ellipse.

The astronomer Johannes Kepler (1571–1630) discovered that the planets travel in elliptical orbits around the sun. Did you know that the moon travels in an elliptical path around Earth and that the rings of Saturn are elliptical, too? So, that's how triangles are related to the motions of the planets and the rings of Saturn!



If you have access to the Internet, you can find out more about Kepler and his laws of planetary motion at the following website:

<http://www.cvc.org/science/kepler.htm>

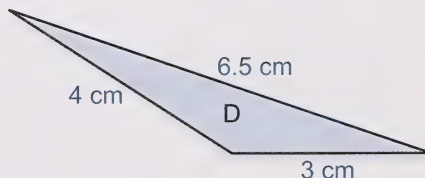
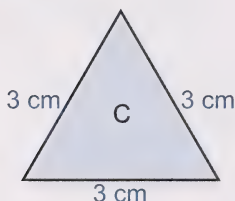
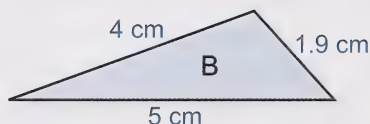
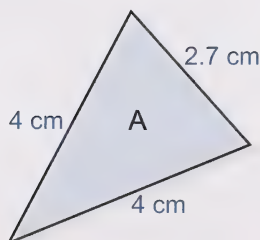
Today you are going to explore triangles. Because the measures of the sides and angles of triangles are so important, you will continue to explore what you know about them.

Triangles can be classified by the lengths of their sides.

Equilateral triangles are triangles that have three sides that are the same length. **Isosceles triangles** are triangles that have two sides that are the same length. **Scalene triangles** are triangles that have no sides the same length.

Example

Which of the following triangles are scalene, equilateral, or isosceles?



Scalene triangles have sides that are different lengths. Triangles B and D are scalene. Isosceles triangles have at least two sides that are the same length. Triangles A and C are both isosceles. However, Triangle C is usually called equilateral because all three sides are the same length.



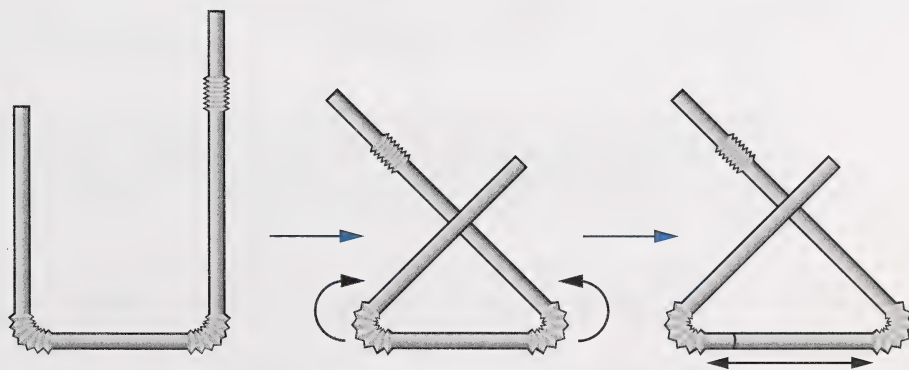
In Lesson 1 of this module, you saw how angles are named according to their size. In this activity you will see that triangles can also be classified by the kinds of angles they have.

You can explore the sides and angles of triangles by connecting three bendable straws. Follow these steps to make triangles that you will use to answer questions 1, 2, and 3.

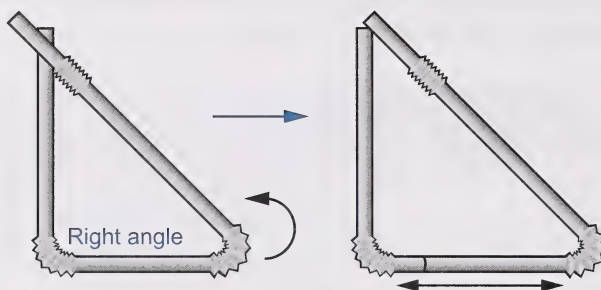
- Cut about 5 cm off the long end of one straw.
- Cut a slit in the short end of the short straw and one of the long straws.
- Pinch the slit ends and slide them inside the long ends to connect the three straws as shown in the following picture.



- You can make different triangles by bending the long straws and/or by making the short straw longer by sliding it.

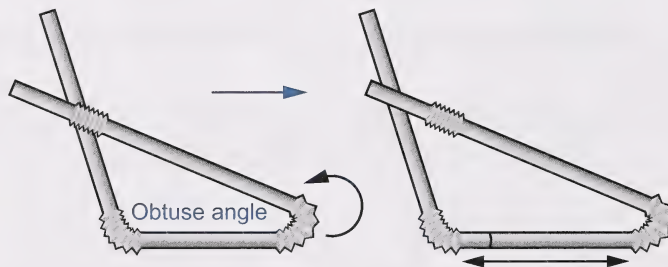


1. Form a right angle with two of the straws. Make triangles by bending the third straw and/or by making the short straw longer by sliding it.



These are called **right triangles** because they have a 90° angle.

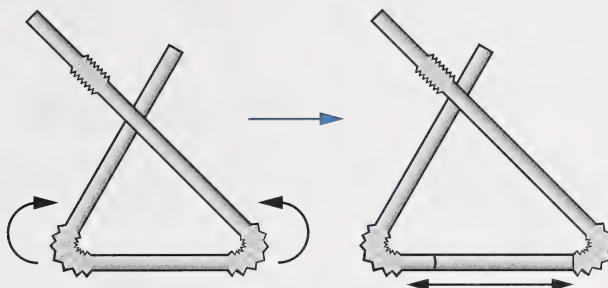
- a. Estimate and then measure the other two angles in each right triangle you made. What kind of angles are they?
 - b. Can the other two angles in a right triangle be any kind other than the ones you named in question 1.a? Explain.
2. Form an obtuse angle with two of the straws. Make triangles by bending the third straw and/or by making the short straw longer by sliding it.



These are called **obtuse triangles** because they have an angle that is greater than 90° .

- a. Estimate and then measure the other two angles in each right triangle you made. What kind of angles are they?
- b. Can the other two angles in an obtuse triangle be any kind other than the ones you named in question 2.a? Explain your thinking.

3. Form an acute angle with two of the straws. Make triangles in which all the angles are acute by bending the third straw and/or by making the short straw longer by sliding it.



These are called **acute triangles** because all of their angles are less than 90° .

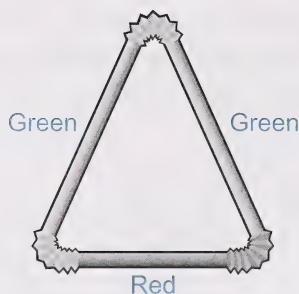
How many angles in a triangle can be acute? Explain.

Check your answers on page 85 in the Appendix.

Use ten bendable straws in each of three different colours (i.e., ten red, ten green, and ten blue). (**Note:** Your straws may not be red, green, and blue; however, for simplicity, the instructions will refer to red, green, and blue straws.)

- Slit the short end of each straw as you did for questions 1 to 3.
- Cut pieces off the red straws so that the long end of each red straw is 10 cm long.
- Cut pieces off the blue straws so that the long end of each blue straw is 7 cm long.
- Do not cut the green straws. (They should stay about 15 cm long.)

To make triangles, join three straws at a time by sliding each slit end completely (to its bend) into a long end.



Make triangles that have as many different sizes and shapes as you can. Use these triangles to answer questions 4 and 5.

4.
 - a. How many triangles of different sizes and shapes is it possible to make?
 - b. For any three straws, is it possible to make more than one shape of triangle? Explain.
 - c. Is there a set of three straws that cannot form a triangle? Explain.
5. Make a large Carroll diagram, similar to the one that follows, on a large sheet of paper (about 60 cm by 60 cm). Place all the triangles you made in the correct cells in the diagram.

	Equilateral	Isosceles	Scalene
Acute			
Right			
Obtuse			

6. Tell what properties a triangle would need to belong in each of the empty cells in the Carroll diagram.

Questions 7 and 8 are optional. Do them if you need more practice.

7. Try to make triangles that belong in the empty cells by cutting the straws to any lengths you choose. Place each one you make in its correct cell.
8. For which empty cells is it not possible to make a triangle? Explain.

Check your answers on pages 85 to 87 in the Appendix.

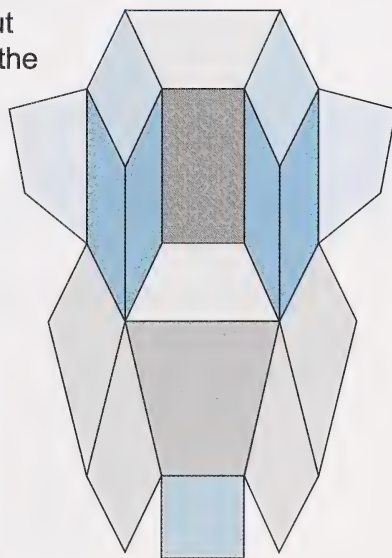
Activity 2



Today you will explore shapes and symmetry.

Loie was using paper pattern pieces to cut out shapes from different colours of cloth to sew the quilt design shown. She knew it would be less work if she could fold the pattern pieces in half and place them on the folded edge of the cloth before cutting them. She found, however, that it was not possible to fold and then cut out all the shapes in this way.

When a shape can be folded in half so that one half fits on top of the other without any overlapping, it has **line symmetry**. The fold line is called the **line of symmetry** or the **mirror line**.



In the Appendix, find the sheet of eight different shapes Loie was using. Cut out the shapes and use them to answer questions 1 to 9.

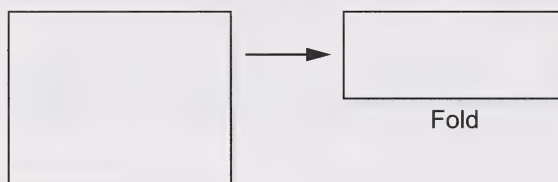
1.
 - a. How are all the shapes the same?
 - b. What name describes this set of shapes?

Check your answers on page 87 in the Appendix.

Use Loie's method of making shapes that is described on the previous page. Begin by trying to fold each shape in half on a line of symmetry. This can be done in more than one way for some shapes and not at all for others. You may wish to verify each fold by placing it on the folded edge of a sheet of paper and cutting out the shape.

Example

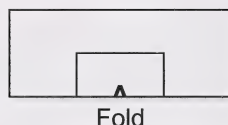
Fold a full sheet of paper in half as shown.



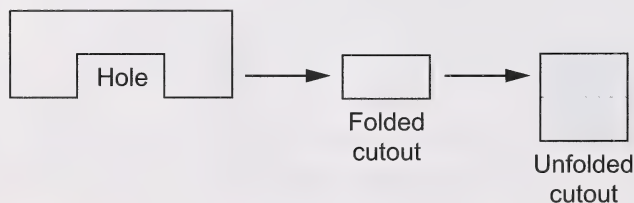
Fold Shape A in half.



Place the folded square along the folded paper.



Cut out the piece from the sheet of paper and unfold it.



2.
 - a. Identify the shapes that have exactly one line of symmetry and name them in as many ways as you can.
 - b. Draw the shapes and show the line of symmetry in each.
3.
 - a. Identify the shapes that have exactly two lines of symmetry and name them in as many ways as you can.
 - b. Draw the shapes and show the lines of symmetry.
4.
 - a. Identify the shape that has four lines of symmetry and name it in as many ways as you can.
 - b. Draw the shape and show the lines of symmetry.
5.
 - a. Which shape can be folded into identical halves but does not have line symmetry?
 - b. Name the shape in as many ways as you can.
 - c. Explain why the fold line is not a line of symmetry.

Check your answers on pages 87 and 88 in the Appendix.

Use eight bendable straws of one colour and twelve of another colour (e.g., eight yellow and twelve blue). (**Note:** Your straws may not be yellow and blue; however, for simplicity, the instructions will refer to yellow and blue straws.)

- Slit the short end of each straw as you did in Activity 1.
 - Cut about half of the long end off each blue straw.
 - Do not cut the yellow straws.
6. Join the straws like you did in Activity 1 to make quadrilaterals like Shapes A, B, C, D, and E from question 1.
 - a. How are the sides of Shape B the same as the sides of Shape E? How are they different?

- b. How are the sides of Shape C the same as the sides of Shape D?
How are they different?
7. Find the straw quadrilateral (Shape A, B, C, D, or E) you can bend to make each of the following shapes from question 1. Use pencil crayons to sketch pictures that show the quadrilaterals you used before and after you bent them.
- a. Shape F b. Shape G c. Shape H

Questions 8 and 9 are optional. Do them if you need more practice.

8. Identify each of the following quadrilaterals and sketch a picture of it. Explain how you decided on your answer.
- a. It has four lines of symmetry and a perimeter of 12 cm.
- b. It has exactly two lines of symmetry, no right angles, and a perimeter of 20 cm.
- c. It has exactly one line of symmetry and no parallel sides. It has a perimeter of 32 m and a side that measures 6 m.
9. Find and list objects in the world around you that have the shapes of at least four different quadrilaterals.

Check your answers on pages 88 to 91 in the Appendix.



Activity 3



Today you will explore polygons.

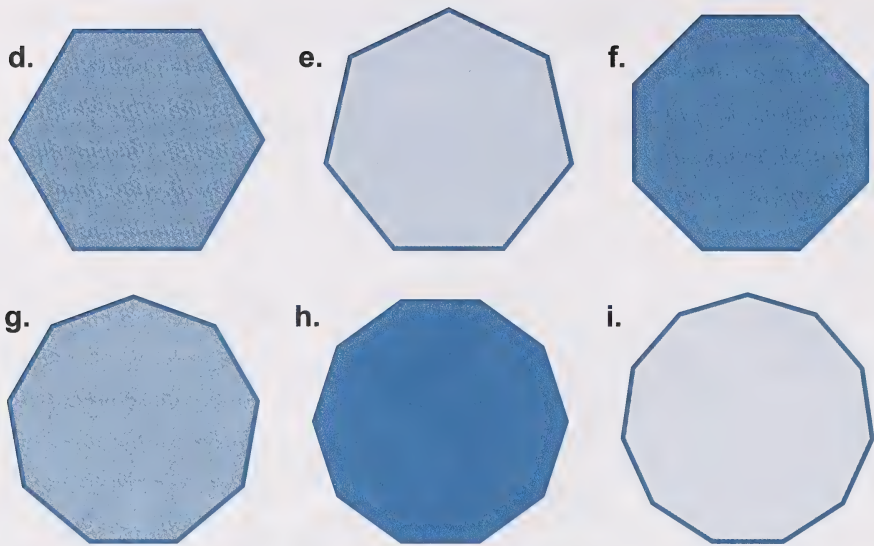


Many traffic signs have unique shapes so that drivers can easily recognize them. Most traffic signs are polygons.

A **polygon** is a closed figure with straight sides. Polygons are named according to the number of sides they have.

1. How many of the following polygons can you identify? Write the names of those you know.





Check your answers on page 91 in the Appendix.

Use 11 bendable straws of the same colour. Slit the short end of each straw as you did in Activity 1. Begin by making a triangle, and then add one straw at a time to make each of the polygons shown in question 1. Each time you make a polygon, see if you can bend it to change the sizes of the angles.

2. If all the sides of a polygon are equal, do all the angles have to be equal? Explain your thinking.

A **regular polygon** is one in which all of the sides are equal and all of the angles are equal.

3. a. In Activity 1, what special name was given to a regular triangle?
b. In Activity 2, what special name was given to a regular quadrilateral?

Check your answers on page 91 in the Appendix.

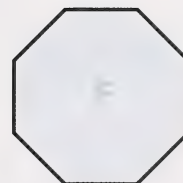
You may wish to refer to this table when you work through the remaining questions in this activity.

You should remember the names in bold. These are the most common polygons that you will encounter.

Number of Sides	Name of the Polygon
3	triangle
4	quadrilateral
5	pentagon
6	hexagon
7	heptagon
8	octagon
9	nonagon
10	decagon
11	hendecagon

Cut out the regular polygons from the Appendix and use them to answer questions 4 to 6.

4. Try to fold each shape in half on a line of symmetry. Do this in as many ways as possible. Draw the lines of symmetry on the following pictures.



5. Copy and complete the following table in your notebook to show the number of lines of symmetry you found.

Polygon	Name	Number of Sides	Number of Lines of Symmetry
A			
B			
C			
D			
E			
F			
G			
H			
I			

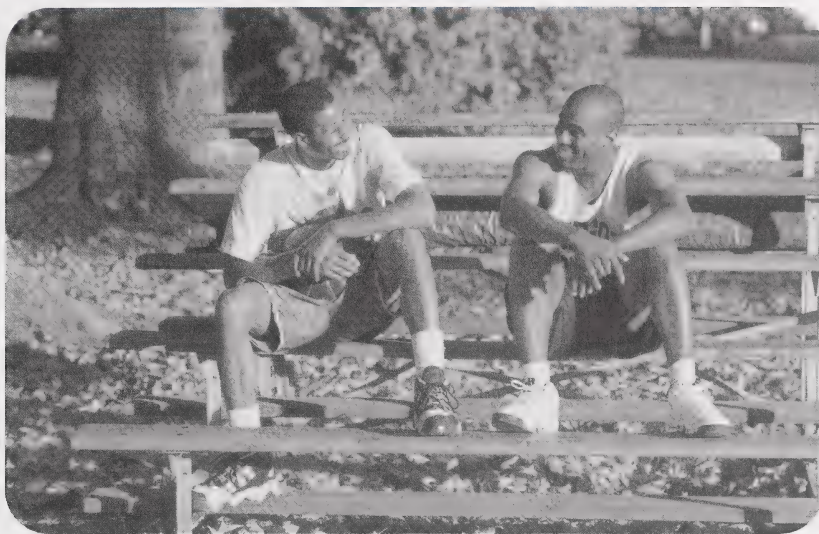
6. What pattern do you see in the chart that relates the number of sides in a regular polygon to the number of lines of symmetry it has?
7. Make a rule for drawing lines of symmetry on regular polygons with an even number of sides.
8. Make a rule for drawing lines of symmetry on regular polygons with an odd number of sides.

Questions 9 and 10 are optional. Do them if you need more practice.

9. Draw the following polygons.
- a. a pentagon that has exactly one line of symmetry
 - b. a hexagon that has exactly two lines of symmetry
10. Find and describe at least four different regular polygons with five or more sides that you may find around you in the world.

Check your answers on pages 91 to 93 in the Appendix.

Challenge Activity



A park is in the shape of a regular polygon. It has less than 12 sides and a perimeter of 120 m. The lengths of all the sides of the park are in whole metres.

1. Sketch pictures of all possible parks and label their dimensions.
2. If the perimeter of the park is 1 m less and its shape is still a regular polygon, it will have three less sides than it does. At the same time, if its perimeter is 1 m greater and its shape is still a regular polygon, it will have 1 more side than it does. What shape is the park?

Check your answers on pages 93 and 94 in the Appendix.

Conclusion

In this lesson you extended what you have learned before about triangles, quadrilaterals, and other polygons. You classified triangles according to the sizes of their angles. You investigated symmetry in polygons and used this property to classify quadrilaterals.



DOUGLAS J. CARDINAL ARCHITECT LTD.

Are you considering a career in architecture? If you are, you will have to learn a great deal about geometric shapes and how they can be incorporated into your designs.

The photograph shows Kainai Middle School on the Blood Reserve at Standoff, Alberta. The school was designed by Douglas J. Cardinal, one of Canada's most famous architects. He has earned worldwide recognition for his work, which incorporates First-Nations themes.



If you have access to the Internet, you can find out more about Douglas Cardinal and his accomplishments at the following website:

http://525.fims.uwo.ca/~sjmiller/site/architecture_dcardinal.html

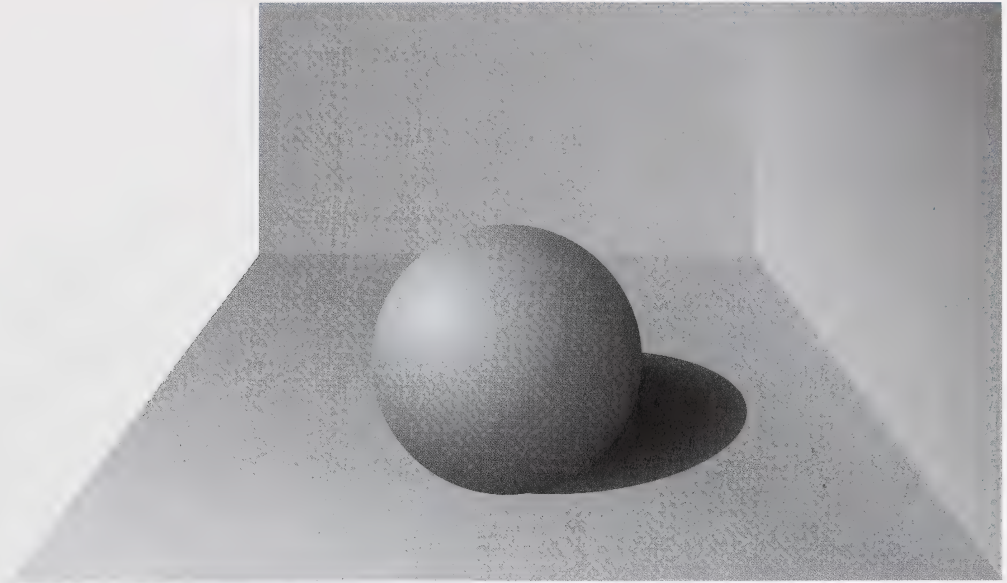
Turn to Assignment Booklet 6A and complete the Lesson 2 Assignment.

When you are done, submit Assignment Booklet 6A to your teacher to be marked.

Lesson 3



Drawing Objects and Shapes



The artist who drew this picture wanted to create an impression of an object seen in three dimensions—an object with depth as well as length and width. Even though you experience the world around you in three dimensions, when you look at objects printed on a page of a book, they are printed in two dimensions only. The artist must try to trick the eye of the person looking at the image!

In this lesson you will explore objects and shapes in two and three dimensions. You will draw pictures and skeletons of 3-D objects using plain paper, oblique grids, and isometric paper. You will use square grid paper to copy 2-D pictures.

Activity 1



Today you will explore objects in three dimensions.

*Space junk is becoming a serious problem.
Even a paint fleck can chip a Space Shuttle
window.*

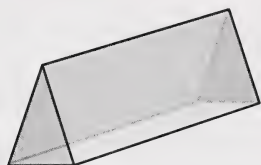


Millions of pieces of space junk have been accumulating in space since *Sputnik* was launched in 1957. Large items include parts of satellites that are no longer in use and hatches that have blown off space modules. Because it travels at tremendous speeds of up to 28 000 km/h, even a tiny piece of debris in space can be dangerous. Radar is used to identify and track this hazardous orbiting space junk.

While on the International Space Station, Commander Claire observed some pieces of space junk. She noted the shapes and sizes of their faces, which she later visualized in order to sketch pictures of them.

1. One piece of space junk was described as a **triangular prism** whose bases were equilateral triangles. The other faces were three times as long as they were wide. Which of the following sketches appears to match this description most closely? Explain why the others do not.

A



B



C



2. Use a pencil crayon to trace the two triangular bases of each prism shown in question 1. Once the bases are drawn, explain how you can complete the picture of each prism.
3. a. What shape are the non-triangular faces in a triangular prism?
- b. Outline the faces from question 3.a. on the pictures of the triangular prisms in question 1 by tracing over them. What shape did you trace that was different than the shapes you named in question 3.a.? Why is this shape different?
4. A small piece of space junk was described as a square prism measuring 2 cm by 2 cm by 5 cm.
- a. Describe all the faces this prism has.
- b. Draw this prism using the method from question 2.
- c. Outline the non-square faces and explain how the shapes you trace compare with the actual shapes they represent.

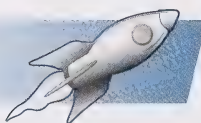
Check your answers on pages 94 and 95 in the Appendix.

5. Another piece of space junk was described as a square prism with some faces that were 3 cm by 4 cm.
- a. Can you be sure what the prism looks like? Explain.
- b. Draw diagrams to support your reasoning.

6. A small piece of space junk was described as a cube with an edge of 3 cm.
 - a. Predict what the shapes the faces will look like in a picture of the cube.
 - b. Draw the cube and verify your predictions.
7. A piece of cylindrical pipe about the size of a soup can was seen orbiting.
 - a. Draw a picture of this object.
 - b. Outline the faces that represent the two bases in your picture. Explain how the shapes you trace compare with the actual shapes they represent.
8. How are 2-D pictures of objects the same as the 3-D objects they represent? How are they different?

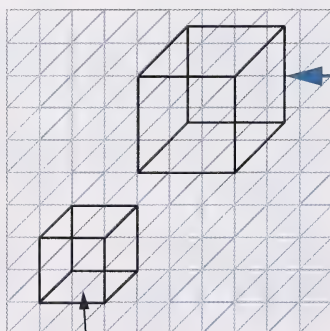
Check your answers on pages 95 and 96 in the Appendix.

Activity 2



Today you will learn how to draw 3-D objects on oblique grid paper, isometric grid paper, and isometric dot paper.

The following picture shows the skeletons of two cubes drawn on an oblique grid.



Base

This is a cube that measures 3 units on each side. It was drawn using three horizontal grid spaces and three vertical grid spaces, but only one-and-a-half diagonal grid spaces.

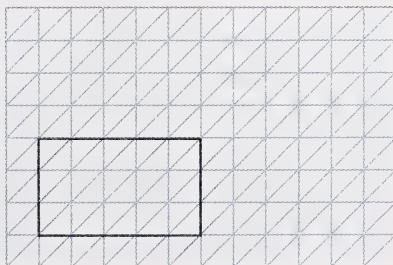
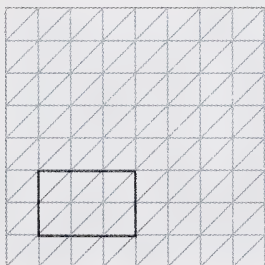
In the drawings, one diagonal grid space represents the same distance as two horizontal or vertical grid spaces. This was done to make your eyes see the drawings as cubes.



1. Explain how using an oblique grid to draw objects is similar to the method used in Activity 1.
2. How is using the grid helpful compared to using plain paper?
3. Measure the edges of the pictures of the cubes and compare their lengths. What do you notice? Explain.
4. The front faces of two rectangular prisms are shown in the following pictures. Complete the drawings. (**Note:** The first two measurements listed for each prism refer to its base.)

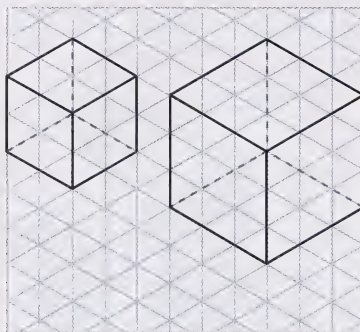
a. 3 units by 4 units by 2 units

b. 5 units by 4 units by 3 units

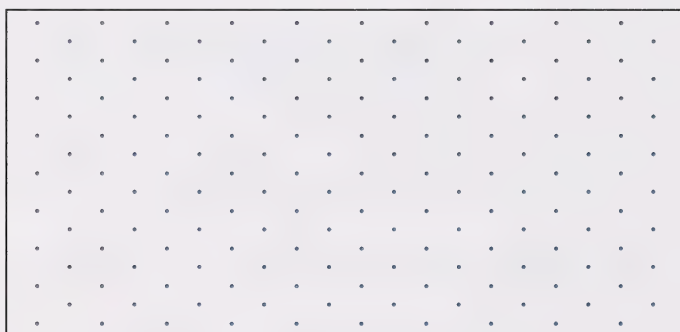


Check your answers on pages 96 and 97 in the Appendix.

The following picture shows the skeletons of a 2 by 2 by 2 cube and a 3 by 3 by 3 cube drawn on an isometric grid.



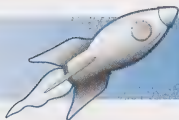
5. Explain how using an isometric grid is different than using an oblique grid.
6. How are the angles formed by the edges of a cube drawn on an isometric grid different from those drawn on an oblique grid?
7. Explain how the following piece of isometric dot paper is similar to an isometric grid.



8. Use a sheet of isometric dot paper from the Appendix to draw the same two rectangular prisms that you drew on the oblique grid in question 4.

Check your answers on page 97 in the Appendix.

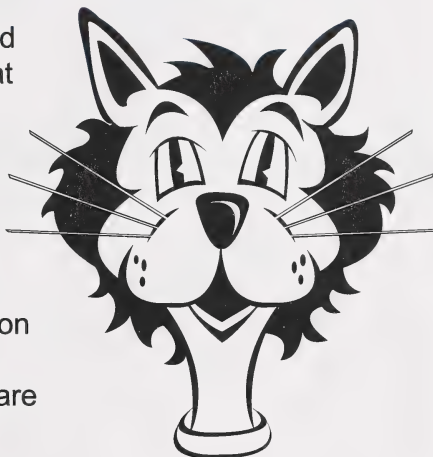
Activity 3



Today you will make larger or smaller two-dimensional drawings with the help of a grid

Leigha wanted to illustrate a poem she had written about cats. She found a picture that she liked and decided to copy it. The picture Leigha found is shown.

She was finding it difficult to copy the picture freehand, so her brother, Liam, suggested that she use a method that he finds helpful. Liam placed an acetate grid on top of the picture and said, “Now you can draw many small pictures, and when you are finished you will have the whole picture.”



1. The picture with the grid placed on top is shown below. Copy the picture, one square at a time, onto a piece of 1.25-cm grid paper from the Appendix. It is important to use a sharp pencil.

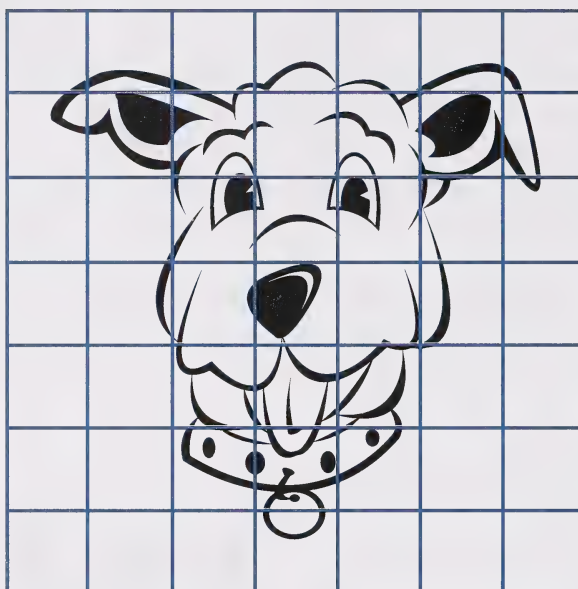


2. After Leigha copied the picture, she was pleased with her work, but she wished the picture was larger. “No problem,” said Liam, “Just draw your picture using a larger grid.”

Use the 2.5-cm grid from the Appendix to draw a larger picture of the cat.

Questions 3 to 5 are optional. Do them if you need more practice.

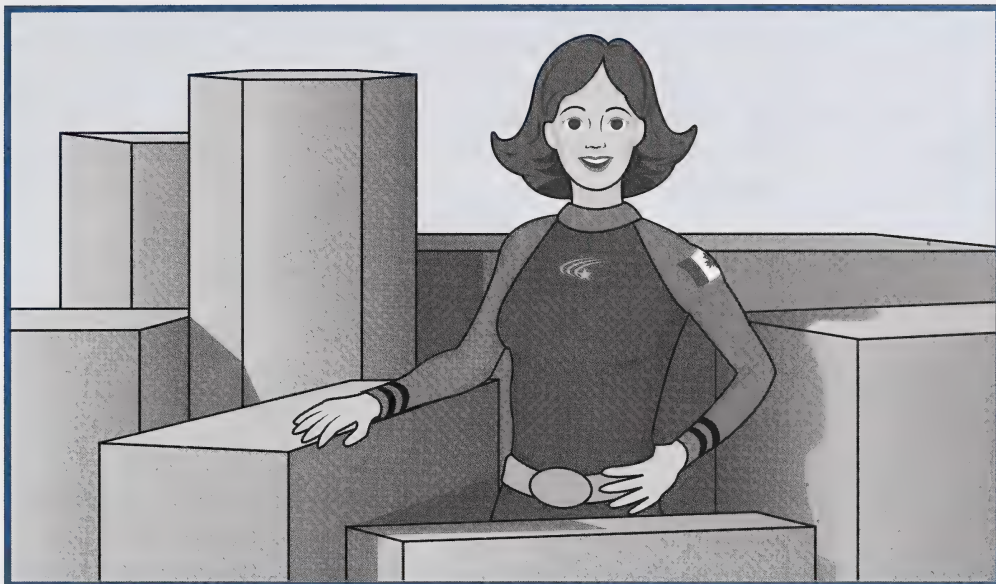
3. To copy a picture of the dog on a smaller grid, use a sheet of 0.5-cm grid paper from the Appendix. Use a sharp pencil.



4. Find a small drawing or picture from a newspaper or magazine. Use a ruler to draw grid lines on it. Draw a larger grid on a different piece of paper and use it to make an enlargement of your drawing or picture.
5. Find a large drawing or picture from a newspaper or magazine. Use a ruler to draw grid lines on it. Draw a smaller grid on a different piece of paper and use it to make a reduction of your drawing or picture.

Check your answers on pages 98 to 100 in the Appendix.

Challenge Activity



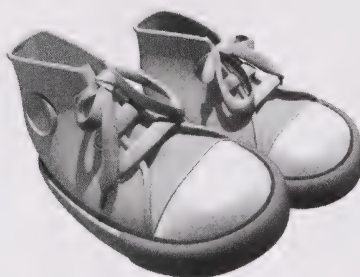
Commander Claire took two storage containers aboard the Space Shuttle. One is a cube and the other is a rectangular prism. Use isometric dot paper to draw all possibilities if the following conditions are true:

- The two containers are equal in volume.
- The area of the base of the rectangular prism is double that of the cube.
- One of the dimensions of the rectangular prism is the same as one of the dimensions of the cube.
- All dimensions are whole numbers of decimetres.
- If you let 1 centicube represent 1 dm^3 , you can build models of both containers using less than 130 centicubes in all.

Check your answers on page 100 in the Appendix.

Conclusion

In this lesson you learned more about drawing objects and shapes. You drew pictures and skeletons of 3-D objects using plain paper, oblique grids, isometric grid paper, and isometric dot paper. You used square grid paper to copy, enlarge, and reduce 2-D pictures.



Are you interested in art? The artist who drew the runners tried to draw a picture that gives you a sense of depth. The artist wants you to “see” the object in three dimensions. See if you can draw this pair of runners in 3-D.

Turn to Assignment Booklet 6B and complete the Lesson 3 Assignment.

Keep Assignment Booklet 6B until you have completed the entire booklet.

Module Summary

In this module you explored angles. You learned how to classify angles, estimate and measure angles, and draw angles.

This module extended your understanding of polygons. You classified triangles according to the sizes of their angles. You investigated symmetry in polygons and used this property to classify quadrilaterals.

You further explored 3-D objects and 2-D shapes. You drew pictures and skeletons of 3-D objects using plain paper, oblique grids, and isometric paper. You used square grid paper to copy 2-D pictures.

You will need your knowledge of angles if you want to determine true north using your magnetic compass. Depending on where you live in Canada, you will have to add or subtract from your compass reading. Near Jasper, Alberta, your compass will point approximately 17° towards the east, away from true north.



Magnetic and true north are different because the North Magnetic Pole is not located at the North Pole. Did you know that the North Magnetic Pole wanders about 10 km every year? Right now it is in Canada's arctic islands, near Ellef-Ringnes Island, Nunavut. If you travelled to the magnetic pole, your compass needle would point straight down!

Turn to Assignment Booklet 6B and complete the Numbers in the News project.

When you are done, submit Assignment Booklet 6B to your teacher to be marked.



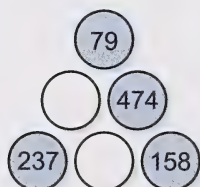
Take out your calculator and complete the following exercises. They will help you review some of the ideas you have learned in Module 6.

Funky Feature: Magic Polygons

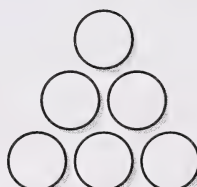
A magic polygon is a regular polygon in which the sum of the numbers on each side is the same (the magic sum). Some of the numbers are missing in the following polygons. For each of the following pairs of magic polygons, use your calculator to help you.

- Find the magic sum and the missing numbers in the first polygon. (Use the least number of keystrokes possible and show your work.)
- Rearrange the same numbers to make a different magic sum in the second polygon. (**Hint:** Simplify the problem by dividing each number in the completed polygon by the least number. Rearrange the resulting numbers to help you make another magic polygon.)

1. a.



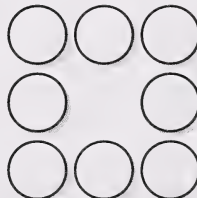
b.



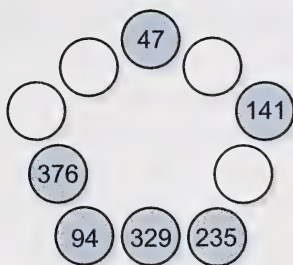
2. a.



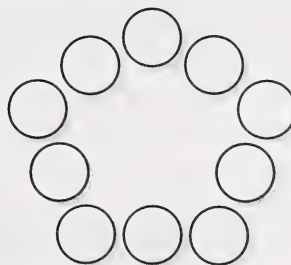
b.



3. a.



b.



Check your answers on pages 101 to 103 in the Appendix.

Funky Feature: Pyramid Power

Serby displays cans in his grocery store by piling them in layers to form pyramids. Layer 1 (the top layer) has one can at the top of the pyramid. The cans in each of the other layers are arranged in a square pattern. A four-layer pyramid is shown.



To find the number of cans Serby uses in any layer, enter the layer number in your calculator, press the **×** the key, and then press the **=** key.

Layer Number	1	2	3	4
Number of Cans in the Layer	1	2 × = 4	3 × = 9	4 × = 16

4. How is the number of cans related to the layer number?
5. The following table shows the keystrokes you can use to find the total number of cans Serby needs to make pyramids with different numbers of layers.

Keystrokes	ON/C	1	M+	2	×	=	M+
Display	0	1	1	2	2	4	4

MR	3	×	=	M+	MR	4	×	=	M+	MR
5	3	3	9	9	14	4	4	16	16	30

Extend the table, and continue using this method up to ten layers. Record your results in a table like the following.

Layer Number	1	2	3	4	5	6	7	8	9	10
Number of Cans in the Layer	1	4	9	16						
Total Number of Cans in the Pyramid	1	5	14	30						

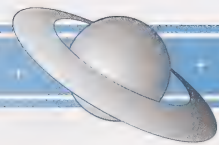
6. Copy and complete the following table to show your keystrokes to find the number of cans in a 16-layer pyramid. (**Hint:** Begin with your answer from question 2 for the number of cans in a ten-layer pyramid.)

Keystrokes				
Display				

7. What is the highest pyramid (greatest number of layers) Serby could make if he had a total of 3000 cans to stack? Explain. Show your keystrokes. (**Hint:** Use your answer to question 3.)

Check your answers on pages 103 and 104 in the Appendix.

Review



The activities in this lesson will help you review and apply what you learned in Module 6 and prepare for the final test. Discuss with your home instructor when you should begin the Review and how much of the Review you should complete.



1. Turn to page 210 of your textbook to On Your Own. Do question 1.
2. Turn to page 210 of your textbook to Practise Your Skills. Do questions 1 to 6.

If you need help with questions 1 and 2, look back at Lesson 1, where you learned about identifying angles. If you feel you need more practice, do question 3.

3.
 - a. Turn to page 222 of your textbook to Skill Bank from This Unit. Do questions 1 and 2.
 - b. Turn to page 247 of your textbook to Skill Bank Looking Back. Do question 4.
 - c. Turn to page 271 of your textbook to Skill Bank Looking Back. Do question 1.

Check your answers on page 105 in the Appendix.



4. Turn to page 211 of your textbook to Finding the Measures of Angles. Do questions 1 and 2.
5. Turn to page 212 of your textbook to Practise Your Skills. Do questions 1 and 2.
6. Turn to page 215 of your textbook to Practise Your Skills. Do questions 1 to 3.

If you need help with questions 4 to 6, look back at Lesson 1, where you learned about measuring angles. If you feel you need more practice, do question 7.



- 7. a.** Turn to page 222 of your textbook to Skill Bank from This Unit. Do questions 3 and 4.
- b.** Turn to page 247 of your textbook to Skill Bank Looking Back. Do questions 5 and 6.
- c.** Turn to page 271 of your textbook to Skill Bank Looking Back. Do questions 2 and 3.

Check your answers on pages 105 and 106 in the Appendix.



- 8.** Turn to page 215 of your textbook to Practise Your Skills. Do questions 4 to 9.

If you need help with question 8, look back at Lesson 1, where you learned about drawing angles. If you feel you need more practice, do question 9.

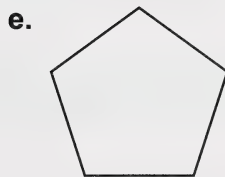
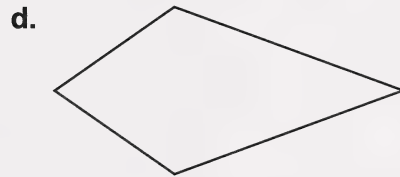
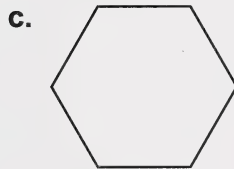
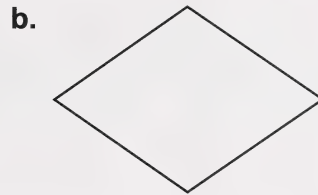
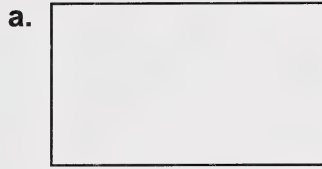
- 9. a.** Turn to page 222 of your textbook to Skill Bank from This Unit. Do question 5.
- b.** Turn to page 271 of your textbook to Skill Bank Looking Back. Do question 4.

Check your answers on pages 107 and 108 in the Appendix.

- 10.** For each of the following five polygons, do the following:

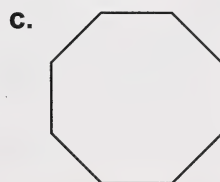
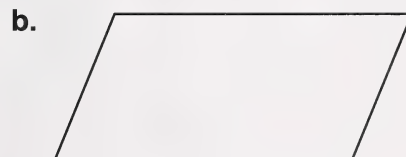
- Name it in as many ways as you can, and tell whether or not it is regular.
- Draw a line to show one way you can divide it into two equal parts.
- Tell whether or not the line you drew is a line of symmetry.

- Identify the shapes made by the line you drew, and write the name inside the shape. Identify each triangle as equilateral, isosceles, or scalene.



If you need help with question 10, look back at Lesson 2, where you learned about polygons. If you feel you need more practice, do question 11.

11. Follow the directions from question 10 for the following three polygons.



Check your answers on pages 108 and 109 in the Appendix.



12. Turn to page 232 of your textbook to Practise Your Skills.

- a. Do questions 1 to 6.
- b. Draw the prism in question 5 on oblique grid paper and on isometric dot paper.

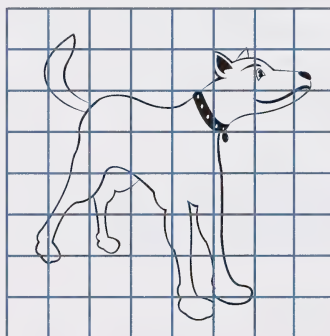
If you need help with question 12, look back at Lesson 3, where you learned about drawing 3-D objects. If you feel you need more practice, do question 13.

13. Turn to page 236 of your textbook to Practise Your Skills.

- a. Do questions 1 to 4.
- b. Draw the prism in question 2 on isometric grid paper and on isometric dot paper.

Check your answers on pages 109 and 110 in the Appendix.

14. Copy the picture of the dog onto 0.5-cm grid paper.



15. If you feel you need more practice, find a simple drawing in a newspaper or magazine. Use a ruler to draw grid lines on it. Draw a blank grid on a different piece of paper, and use it to copy your drawing.



16. Turn to pages 220 to 221 of your textbook to Problem Bank. Do questions 1, 2, 3, 4, and 6.

17. Turn to page 244 of your textbook to Problem Bank. Do question 2.

Check your answers on pages 110 to 114 in the Appendix.



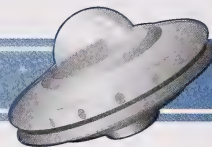
If you need additional work to master the material in this module, work through Lesson 16: Working with Angles on the Mathematics 6 Companion CD.

After completing this lesson, you can print an assignment by clicking on the Activity button at the bottom of the page.

Ask your home instructor to print the solutions to the questions in the activity by clicking on the Parent Notes button at the bottom of the page. Discuss your answers with your home instructor.



Just the Facts



Ask your home instructor to time you as you complete the following timed drill. Your goal is to complete all 25 questions in two minutes. At the end of two minutes, count how many questions you were able to complete. Then use the Answer Key in the Appendix to mark the drill, and record your score in the space provided. Before you move on, go back and complete any questions you did not finish.

Multiplication and Division Facts

$1 \times 5 =$	$12 \div 2 =$	$\begin{array}{r} 3 \\ \times 2 \\ \hline \end{array}$	$4 \overline{)32}$	$8 \times 6 =$
$40 \div 8 =$	$\begin{array}{r} 2 \\ \times 7 \\ \hline \end{array}$	$5 \overline{)5}$	$4 \times 1 =$	$56 \div 8 =$
$\begin{array}{r} 1 \\ \times 0 \\ \hline \end{array}$	$7 \overline{)14}$	$5 \times 9 =$	$36 \div 9 =$	$\begin{array}{r} 7 \\ \times 3 \\ \hline \end{array}$
$6 \overline{)54}$	$0 \times 5 =$	$0 \div 6 =$	$\begin{array}{r} 4 \\ \times 9 \\ \hline \end{array}$	$3 \overline{)9}$
$8 \times 8 =$	$5 \div 1 =$	$\begin{array}{r} 8 \\ \times 6 \\ \hline \end{array}$	$1 \overline{)2}$	$5 \times 0 =$

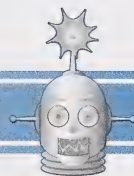
Multiplication and Division Facts

Number completed in 2 minutes:

Number correct in 2 minutes:

Record your score on the Just the Facts Progress Chart.

Mental Math



Suppose you have only \$70 to buy groceries with. You need to keep an approximate running total of how much you have spent.

This tip shows you a different way to use front-end and compensation strategies that are helpful when you need to find the sum of many prices. A quick way to keep a running estimate of the total is to keep adding the dollar amounts only, but to compensate for the low estimate that would result by rounding the dollar amount up to the next higher dollar in every second price.

Example 1

Estimate the running total of the following prices, as you put the items in your shopping cart: \$5.69, \$2.35, \$4.15, \$4.98, \$1.00, \$5.29, \$2.98, \$8.79, \$3.49, \$3.56, \$7.79, \$3.50, \$1.49, \$5.98, \$2.29, and \$6.45.

The running total will be $5 + 3 = 8$, $+ 4 = 12$, $+ 5 = 17$, $+ 1 = 18$, $+ 6 = 24$,
 $+ 2 = 26$, $+ 9 = 35$, $+ 3 = 38$, $+ 4 = 42$, $+ 7 = 49$, $+ 4 = 53$, $+ 1 = 54$,
 $+ 6 = 60$, $+ 2 = 62$, $+ 7 = 69$.

The estimated bill is \$69, and the exact total is \$69.78. That's pretty close! This strategy can be used for estimating the sum of many numbers in other situations.



Example 2

Estimate the total number of books borrowed from the library last year by the 25 members of a book club: 25, 39, 42, 81, 59, 55, 32, 90, 44, 18, 37, 82, 56, 83, 29, 58, 22, 31, 59, 75, 34, 50, 18, 48, and 39.



The total will be $20 + 40 = 60$, $+ 40 = 100$, $+ 90 = 190$, $+ 50 = 240$,
 $+ 60 = 300$, $+ 30 = 330$, $+ 100 = 430$, $+ 40 = 470$, $+ 20 = 490$, $+ 30 = 520$,
 $+ 90 = 610$, $+ 50 = 660$, $+ 90 = 750$, $+ 20 = 770$, $+ 60 = 830$, $+ 20 = 850$,
 $+ 40 = 890$, $+ 50 = 940$, $+ 80 = 1020$, $+ 30 = 1050$, $+ 60 = 1110$,
 $+ 10 = 1120$, $+ 50 = 1170$, $+ 30 = 1200$.

The estimated total is 1200 books, and the exact total is 1206 books!

Try to practise this strategy whenever it is appropriate to use it.

Mathematics 6

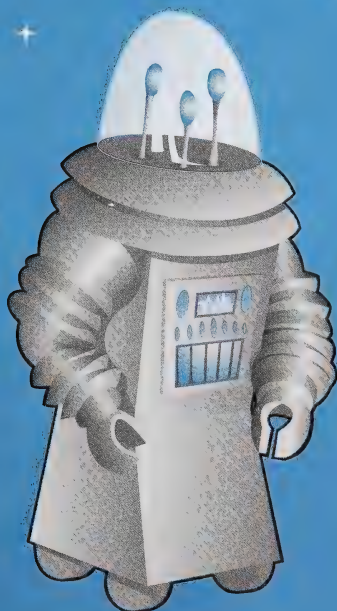
Appendix

Glossary

Answer Key

Image Credits

Learning Aids



Glossary

acute angle: an angle between 0° and 90° in size

acute triangle: a triangle that has three acute angles

arm of an angle: each of the two rays that form an angle

decagon: a ten-sided polygon

ellipse: an oval shape

equilateral triangle: a triangle with three equal sides

hendecagon: an eleven-sided polygon

heptagon: a seven-sided polygon

hexagon: a six-sided polygon

isosceles triangle: a triangle with two equal sides

line of symmetry: a line about which a figure can be folded into two matching halves

mirror line: a line of symmetry

nonagon: a nine-sided polygon

obtuse angle: an angle between 90° and 180° in size

obtuse triangle: a triangle that has one obtuse angle

octagon: an eight-sided polygon

parallelogram: a quadrilateral with opposite sides parallel

pentagon: a five-sided polygon

polygon: a closed two-dimensional figure with three or more sides

protractor: an instrument used to measure angles

quadrilateral: a four-sided polygon

reflex angle: an angle greater than 180° in size

regular polygon: a polygon with equal sides and equal angles

right angle: an angle 90° in size

right triangle: a triangle that has one right angle

scalene triangle: a triangle with no equal sides

side of an angle: an arm of an angle

square: a regular quadrilateral

straight angle: an angle 180° in size

trapezoid: a quadrilateral with two parallel sides

triangle: a three-sided polygon

triangular prism: a prism with triangular bases

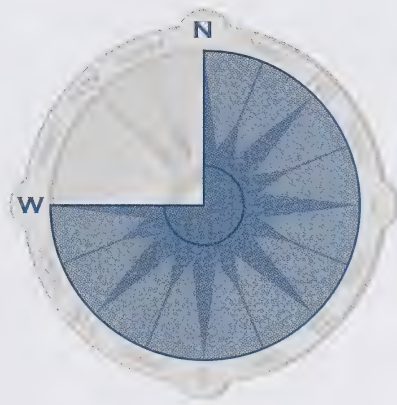
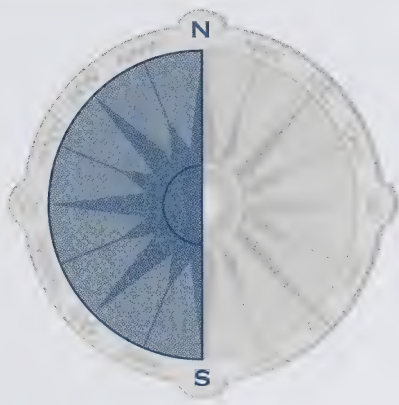
vertex: the common endpoint of two rays forming an angle

Answer Key

Lesson 1: Angles

Activity 1

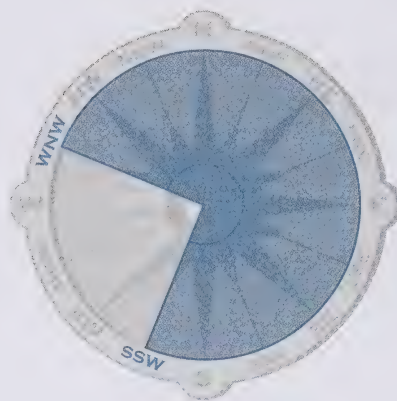
1. a. equal to 180°
c. greater than 90° but less than 180°
e. less than 90°
b. equal to 90°
d. equal to 90°
f. greater than 180°
2. a. straight angle
d. right angle
b. right angle
e. acute angle
c. obtuse angle
f. reflex angle
3. a. straight
b. reflex



c. reflex



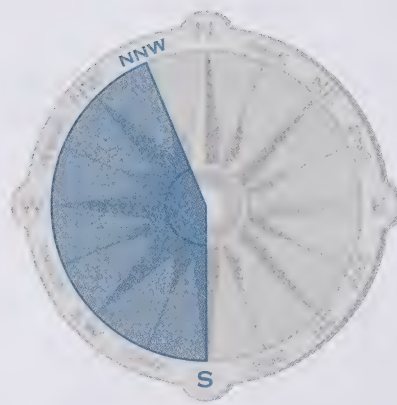
d. reflex



e. reflex



f. obtuse



4. Two different angles can be formed by any two rays because you can go either clockwise or counterclockwise from one ray to the other.

5. a. a reflex angle

b. a straight angle

c. a reflex angle

d. a reflex angle

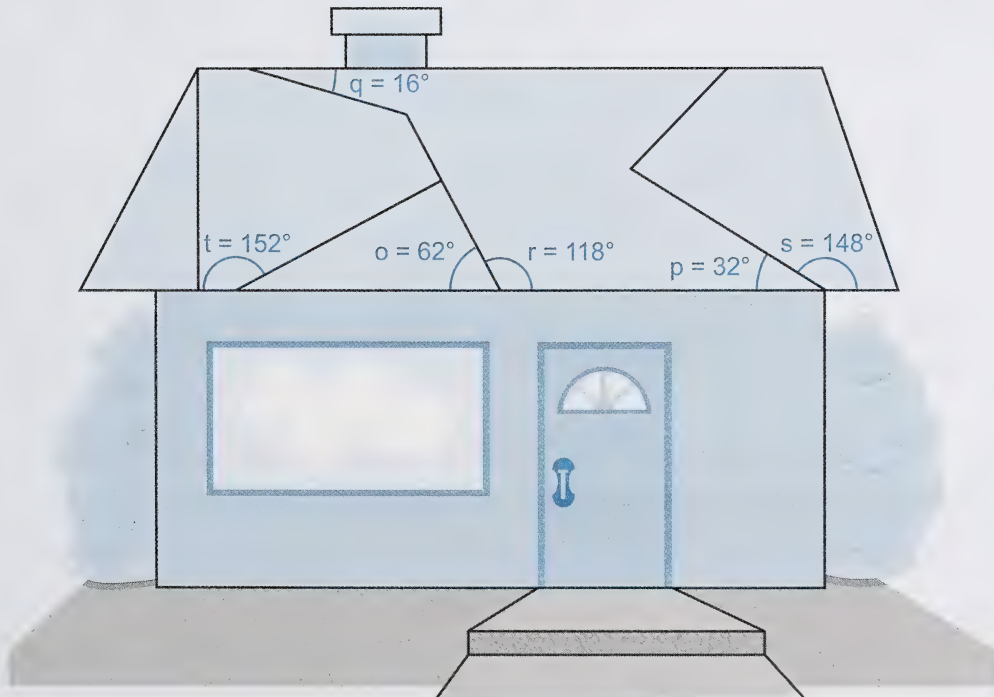
e. It depends on the size of the reflex angle. The other angle could be an acute angle, a right angle, or an obtuse angle.

6.
 - a. SSE, S, SSW, WNW, W, and WSW rays
 - b. all the rays except the NNE ray
 - c. WSW ray
 - d. WSW, SW, SSW, WNW, NW, and NNW rays
 - e. NW and SE rays
7. The time shown is 6:00. The hands form a straight angle.
8.
 - a. an acute angle and a reflex angle
 - b. an obtuse angle and a reflex angle
 - c. a right angle and a reflex angle
9.
 - a. 3:00 and 9:00
 - b. 1:00, 2:00, 10:00, and 11:00
 - c. 4:00, 5:00, 7:00, and 8:00

Activity 2

1.

a. 90°	b. 180°	c. 135°	d. 282°	e. 78°
f. 109°	g. 289°	h. 90°	i. 28°	j. 180°
k. 61°	l. 90°	m. 270°	n. 126°	
- 2.



3. The minute hand moves 360° in 1 h (one complete revolution).

4. a. The hour hand moves 30° in 1 h.

b. Divide 360° by 12, the number of hours in one complete revolution.

$$360^\circ \div 12 = 30^\circ$$

c. It takes the minute hand 5 min to move the same number of degrees.

5. a. The minute hand moves 120° in 20 min.

b. $4 \times 5 \text{ min} = 20 \text{ min}$, so $4 \times 30^\circ = 120^\circ$

6. The minute hand moves 6° in 1 min.

$$1 \text{ h} = 60 \text{ min}, \text{ so } 360^\circ \div 60 = 6^\circ.$$

7. a. It would take about 7 or 8 min for the minute hand to move 45° .

b. Answers will vary. Sample answers are given.

- Using the answer from question 6, $45^\circ \div 6^\circ = 7.5 \text{ min}$.

OR

- It takes 15 min to move 90° . Since $45^\circ = 90^\circ \div 2$, the time is $15 \text{ min} \div 2 = 7.5 \text{ min}$.

8. a. The minute hand moves 210° .

b. Answers will vary. Sample answers are given.

- The minute hand travels for 35 minutes. 30 min is a straight angle (180°), plus the angle formed in 5 min $= 30^\circ$ (from question 4.c.). Therefore, add $180^\circ + 30^\circ = 210^\circ$.

OR

- $7 \times 5 \text{ min} = 35 \text{ min}$ and $7 \times 30^\circ = 210^\circ$.

9. The hands would form a 60° angle at 2:00 and at 10:00.

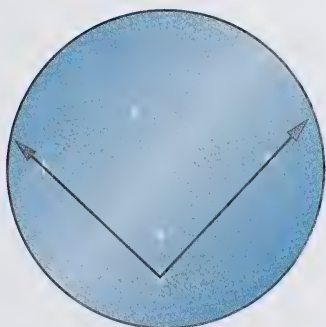
10. a. The hour hand moves about 10° in 20 min.

b. $1\text{ h} = 60\text{ min}$, so 20 min is one-third of an hour ($60 \div 20 = 3$).

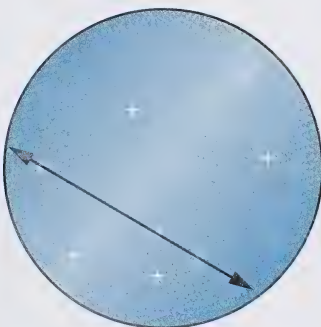
From question 4.a., the hour hand moves 30° in 1 h, so one-third of $30^\circ = 30^\circ \div 3 = 10^\circ$.

Activity 3

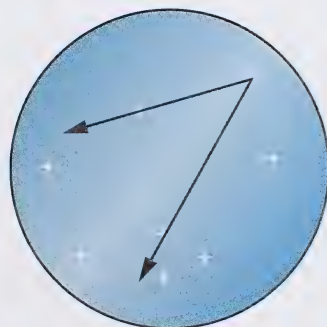
1. right angle



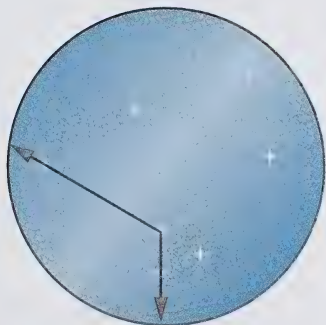
2. straight angle



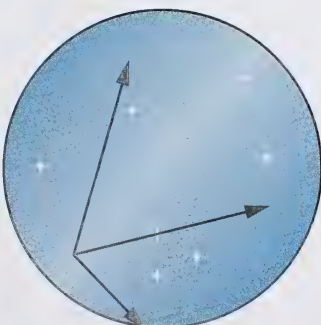
3. 45° angle



4. 120° angle

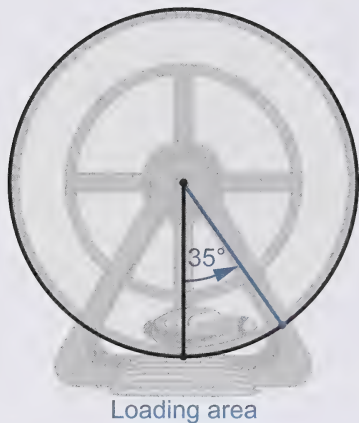


5. 60° angles

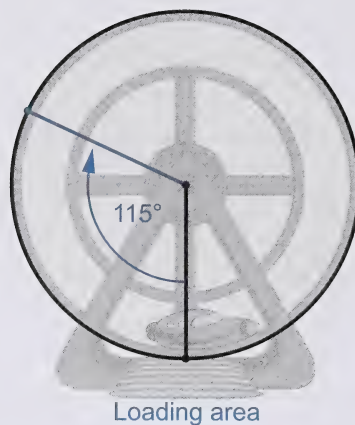


The dots can be any points on the rays.

6. 35° counterclockwise



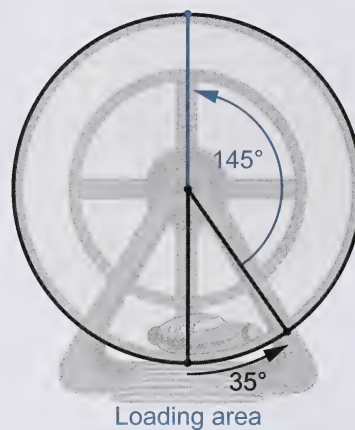
7. 115° clockwise



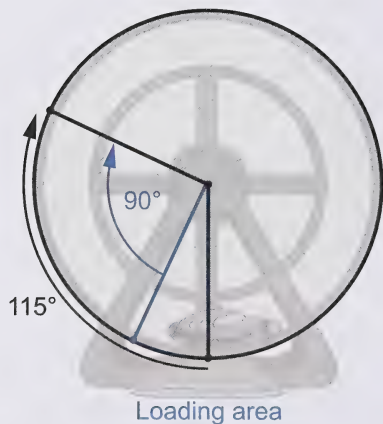
8. 290° clockwise



9. 145° counterclockwise



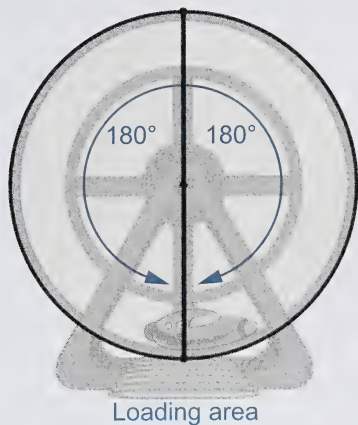
10. 90° counterclockwise



11. 350° clockwise



12. a. 180° either way



b. 25° counterclockwise



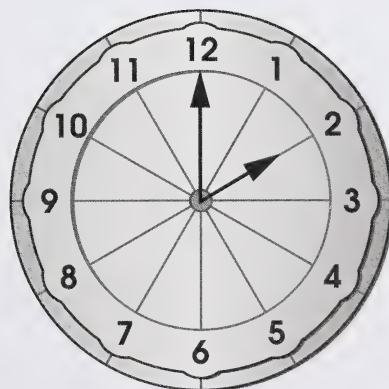
c. 80° clockwise



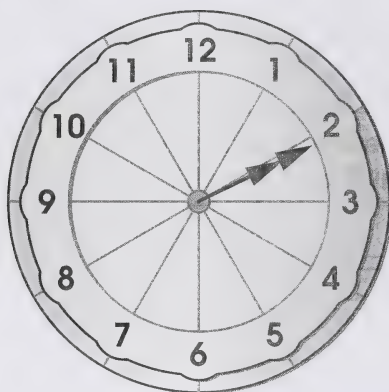
Challenge Activity

The hands will form a 160° angle at 2:40. Strategies will vary. A sample strategy is given.

At 2:00, the hands form a 60° angle. Between 2:00 and 3:00, the hour hand will turn 30° and the minute hand will turn 360° .



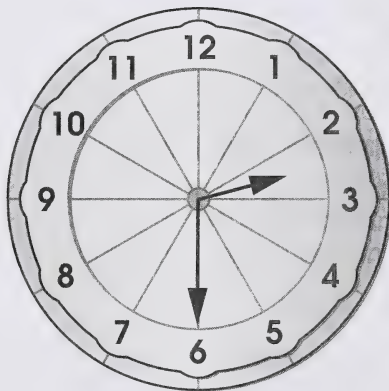
The 60° angle formed by the hands at 2:00 will decrease until the minute hand passes the hour hand between 2:10 and 2:15. After that, the angle formed by the hands increases.



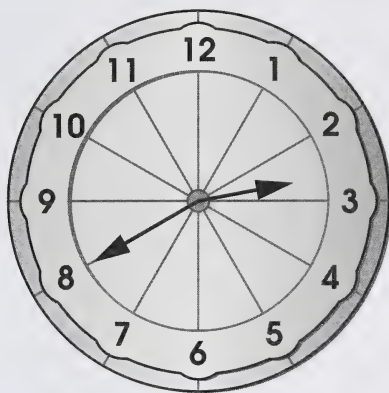
The angle formed by the hands at 2:30 is 105° . (In 1 h, the hour hand moves 30° , so in half an hour, it moves half of 30° , which is 15° . $15^\circ + 90^\circ = 105^\circ$)

OR

From the 12 position, $180^\circ - 75^\circ = 105^\circ$. The hour hand moved $30^\circ + 30^\circ + 15^\circ$, and the minute hand moved 180° .



The angle formed by the hands at 2:40 is $180^\circ - 20^\circ = 160^\circ$. (In 1 h, the hour hand moves 30° , so in one-third of an hour (20 min), it moves one-third of 30° , which is 10° ; 40 min is two-thirds of an hour, so in 40 min it moves $2 \times 10^\circ = 20^\circ$.)



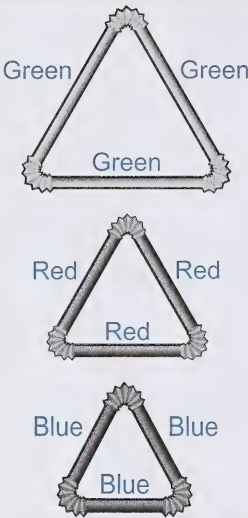
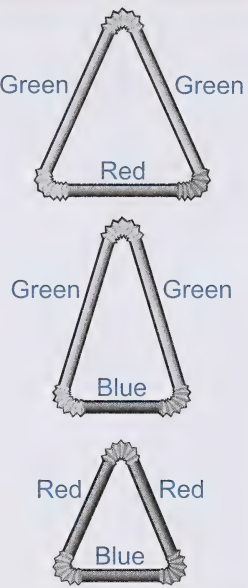
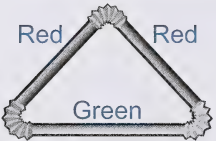
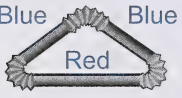
Lesson 2: Polygons

Activity 1

1. **a.** Answers for actual angle measures will vary. The angles are both acute.
b. No, both of the other two angles in a right triangle must be acute. Otherwise, the three sides would not close to form a triangle.
2. **a.** Answers for actual angle measures will vary. The angles are both acute.
b. No, both of the other two angles in an obtuse triangle must be acute. Otherwise, the three sides would not close to form a triangle.
3. In a triangle, two or three angles can be acute.
 - If the triangle has a right angle or an obtuse angle, then both of the other two angles must be acute.
 - All three angles can be acute.
4. **a.** It is possible to make nine triangles of different sizes and shapes.

- b. No, for any three straws, is it only possible to make one shape of triangle no matter how you try to rearrange the straws. (**Note:** If you think you were able to make different triangles from the same set of three straws, then use turns or flips to discover that, in fact, they are the same.)
- c. For the set of two 7-cm straws and one 15-cm straw, it is not possible to make a triangle. The two 7-cm straws only have a total length of 14 cm, so they cannot form a closed figure with a 15-cm straw.

5.

	Equilateral	Isosceles	Scalene
Acute			
Right			
Obtuse			

6. **Acute scalene triangle:** All three angles would have to be less than 90° . The triangle would have to be made with straws of three different lengths (colours).

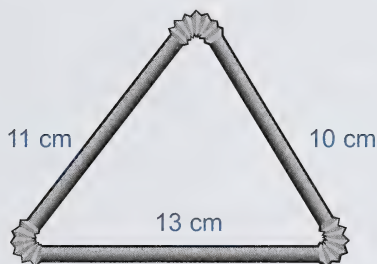
Right equilateral triangle: One angle would have to be equal to 90° . The triangle would have to be made with three straws of the same length (colour).

Right scalene triangle: One angle would have to be equal to 90° . The triangle would have to be made with three straws of three different lengths (colours).

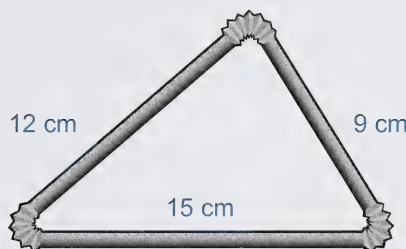
Obtuse equilateral triangle: One angle would have to be greater than 90° . The triangle would have to be made with three straws of the same length (colour).

7. Answers will vary. Sample answers are given.

Acute scalene triangle:



Right scalene triangle:



8. It is not possible to make a right equilateral triangle or an obtuse equilateral triangle. The angles in an equilateral triangle are all equal acute angles. (Each angle is 60° .)

Activity 2

1. a. All of the shapes are polygons with four sides.
b. Polygons with four sides are called quadrilaterals.
2. a. The shapes that have exactly one line of symmetry are Shape C, Shape D, and Shape E. Shape C can be called a quadrilateral or a trapezoid. Shape D can be called a quadrilateral or a trapezoid. Shape E can be called a quadrilateral or a kite.

b. Shape C



Shape D



Shape E



3. a. The shapes that have exactly two lines of symmetry are Shape B and Shape H. Shape B can be called a quadrilateral, parallelogram, or rectangle. Shape H can be called a quadrilateral, parallelogram, or rhombus.

b. Shape B



Shape H



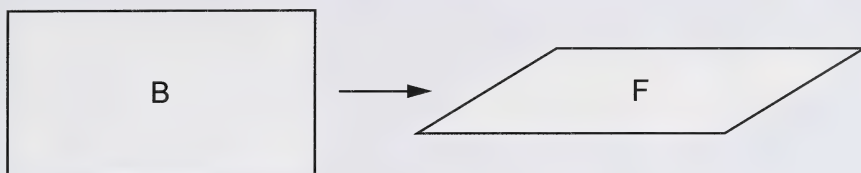
4. a. The pattern shape that has four lines of symmetry is Shape A. Shape A can be called a quadrilateral, parallelogram, rectangle, rhombus, or square. If you are unsure of any of these terms, look up their definitions in the glossary.

b.

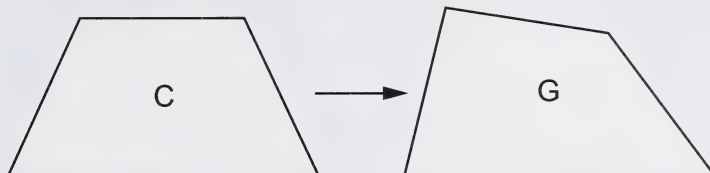


5. a. Shape F can be folded into identical halves, but it does not have line symmetry.
- b. Shape F can be called a quadrilateral or a parallelogram.
- c. The fold line is not a line of symmetry because when you fold the shape, the halves do not overlap each other completely.
6. a. The sides of Shape B are the same lengths as the sides of Shape E. In Shape B, the longer sides are opposite each other; and in Shape E, the longer sides touch (are adjacent to) each other.
- b. Both Shape C and Shape D have three equal sides. In Shape C the three equal sides are shorter than the fourth side, and in Shape D the three equal sides are longer than the fourth side.

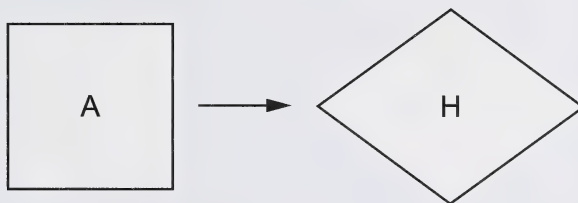
7. a. Quadrilateral B can bend to make Shape F.



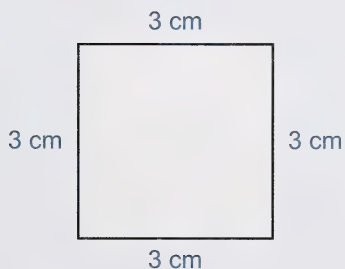
- b. Quadrilateral C can bend to make Shape G.



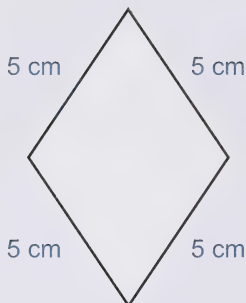
- c. Quadrilateral A can bend to make Shape H.



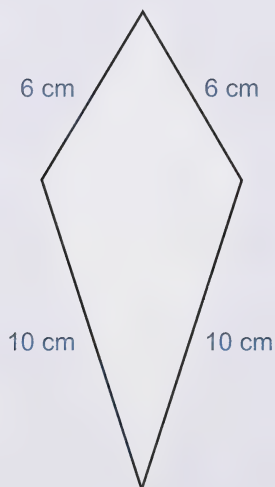
8. a. It is a square because a square is the only quadrilateral with four lines of symmetry. Its sides are $12 \text{ cm} \div 4 = 3 \text{ cm}$ because the perimeter of 12 cm is the sum of four equal sides.



- b. It is a rhombus. The only quadrilaterals that have exactly two lines of symmetry are rectangles and rhombuses, and since it has no right angles, it cannot be a rectangle. Its sides are 5 cm each because the perimeter is the sum of four equal sides, and $20 \text{ cm} \div 4 = 5 \text{ cm}$.



- c. It is a kite. Trapezoids have exactly one line of symmetry, but they have parallel sides. It has two sides of 6 m each and two sides of 10 m each. If one side is 6 m, then it must have another side that is 6 m. The total length of those two sides is 12 m. Since the perimeter is 32 m, the other two sides must have a total length of $32 \text{ m} - 12 \text{ m} = 20 \text{ m}$. The other two sides are equal, so each is $20 \text{ m} \div 2 = 10 \text{ m}$.



9. Answers will vary. Sample answers are given.

squares: ceramic tiles, windows

parallelograms: designs in stained glass, unfolded surface of frozen juice can (cardboard cylinder)

rectangles: doors, windows, place mats, pages in a book

trapezoids: tables, pie-shaped house lot, end panel of cardboard filing container

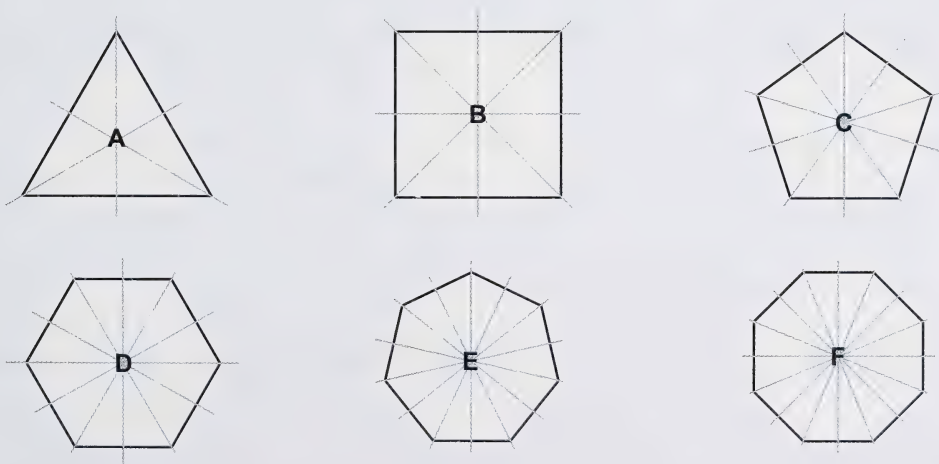
rhombuses: end panel of designer box

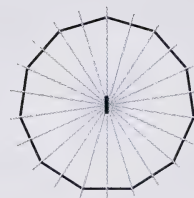
kites: kite

Activity 3

- triangle
 - quadrilateral
 - pentagon
 - hexagon
 - heptagon
 - octagon
 - nonagon
 - decagon
 - hendecagon
- Except for a triangle, if all the sides of a polygon are equal, the angles do not have to be equal. You were able to bend the straw polygons into many different shapes.
- In Activity 1, you learned that a regular triangle is called an equilateral triangle.
 - In Activity 2, you learned that a regular quadrilateral is called a square.

4.





5.

Polygon	Name	Number of Sides	Number of Lines of Symmetry
A	triangle	3	3
B	quadrilateral	4	4
C	pentagon	5	5
D	hexagon	6	6
E	heptagon	7	7
F	octagon	8	8
G	nonagon	9	9
H	decagon	10	10
I	hendecagon	11	11

6. The number of lines of symmetry a regular polygon has is the same as the number of sides it has.
7. On regular polygons with an even number of sides, you draw lines that connect the opposite vertices and lines that connect the midpoints of the opposite sides.
8. On regular polygons with an odd number of sides, you draw lines that connect the vertices to the midpoints of the opposite sides. (The lines of symmetry meet the sides at right angles.)
9. Answers will vary. Sample answers are given.
- a. pentagon with one line of symmetry b. hexagon with two lines of symmetry



10. Answers will vary. Sample answers are given.

pentagons: Chrysler logo, the Pentagon building in the United States

hexagons: honeycombs, snowflakes, fancy windows, the inside area of some Chinese Checkers boards

heptagons: base of a week-at-a-time pill box

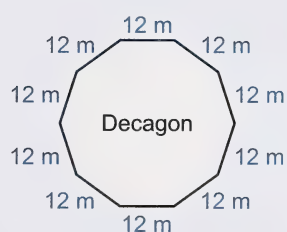
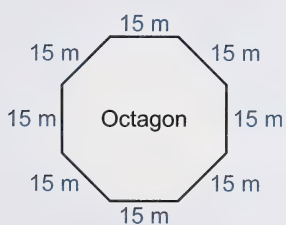
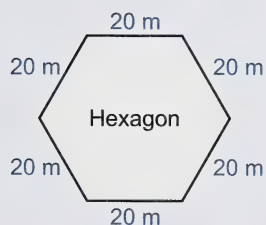
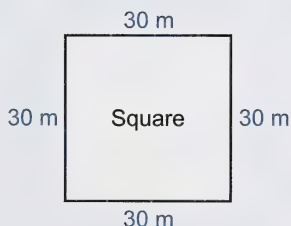
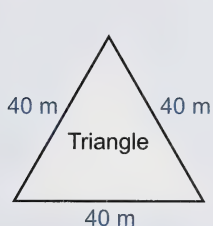
octagons: stop signs, some hot tubs

decagons: some gazebos

hendecagons: loonie, some screen houses

Challenge Activity

1. There are six possible regular polygon shapes for the park.



2. The park is in the shape of a regular decagon.

- If the perimeter of the park is 1 m less, it will be $120\text{ m} - 1\text{ m} = 119\text{ m}$.
If the perimeter of the park is 1 m more, it will be $120\text{ m} + 1\text{ m} = 121\text{ m}$.
- Find a number less than 12, such that 3 less than it divides 119 evenly and 1 more than it divides 121 evenly.

Since 119 is odd, its factors must be odd.

- Try 3. $119 \div 3 = 39.666\ 666$ (no)
 - It can't be 5 because multiples of 5 end in 5 or 0.
 - Try 7. $119 \div 7 = 17$ (Yes, a seven-sided park with all sides 17 m has a perimeter of 119 m. Because it has three sides more than this, the park has ten sides.)
- Test to see if 1 more than 10 divides 121 evenly: $121\text{ m} \div 11 = 11\text{ m}$. (Yes, an 11-sided park with all sides 11 m has a perimeter of 121 m.)

Lesson 3: Drawing Objects and Shapes

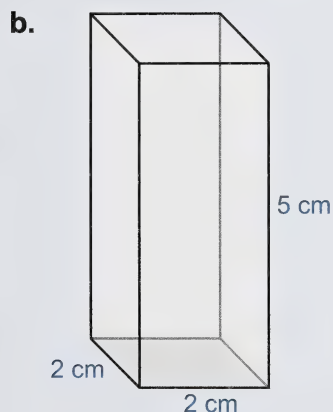
Activity 1

1. Sketch C matches this description.

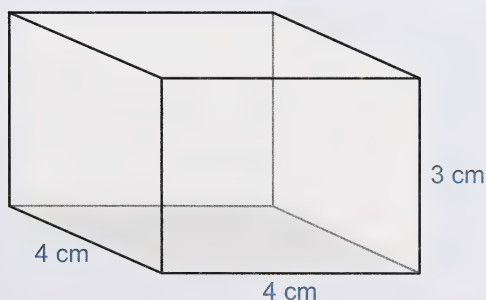
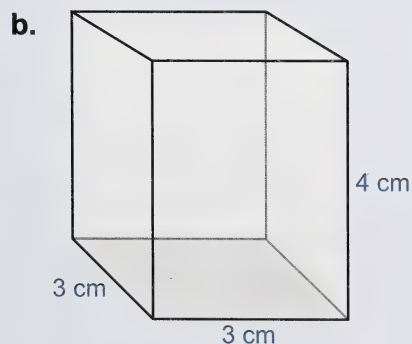
- Sketch A does not have bases that are equilateral triangles. (They are isosceles triangles.)
- Sketch B looks like its rectangular faces are 4 or 5 times longer than they are wide.

2. Once the bases are drawn, connect each vertex of the first triangle to its matching vertex of the second triangle. The lines drawn will be parallel to each other. The edges that are behind a face can be shown as dotted lines.

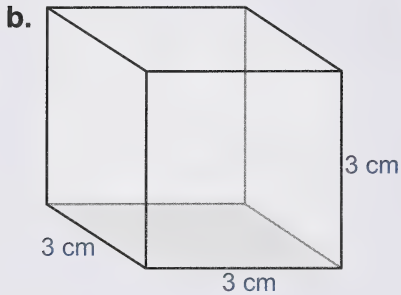
3. a. The non-triangular faces in a triangular prism are rectangles.
- b. Two of the outlined shapes are parallelograms. Parallelograms may be drawn to represent rectangles to give a 3-D effect to drawings on a 2-D (flat) sheet of paper.
4. a. The prism will have two square faces, each measuring 2 cm by 2 cm. It will also have four rectangular faces, each measuring 2 cm by 5 cm.



- c. When you trace the non-square faces, you will discover that the two on the left and right sides are parallelograms, but they represent rectangles.
5. a. You can't be sure of the size of the prism because it could have two squares measuring 3 cm by 3 cm and four rectangles measuring 3 cm by 4 cm; or two squares measuring 4 cm by 4 cm and four rectangles measuring 4 cm by 3 cm.

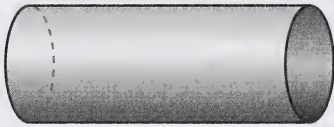


6. a. Some of the faces will look like rhombuses in the picture.



Yes, all but the front and back faces look like rhombuses.

7. a. The cylindrical pipe might look like the following.

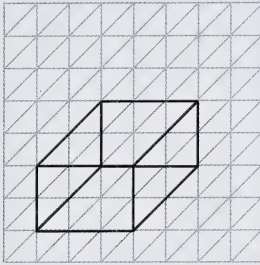


- b. The two bases you trace are ovals, but they represent circles.
8. The 2-D pictures have the same number of parts (faces) as the 3-D objects they represent, but some of the shapes are different to give the drawing perspective (depth). Opposite edges that are parallel in the object stay parallel in the picture.

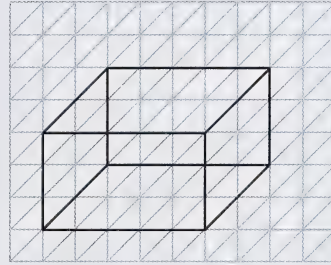
Activity 2

1. Using an oblique grid to draw objects is similar to the method used in Activity 1 because you start by drawing the front face, and then you draw the back face. Third, you connect matching vertices by drawing lines that are parallel to each other.
2. Using the oblique grid is helpful because the parallel lines are already drawn in.
3. The diagonal edges of the cubes in the drawing are shorter than its vertical and horizontal edges. Yet, one diagonal grid space is longer than one vertical or horizontal grid space. Therefore, to make the drawing look like a cube, fewer grid spaces are used to draw the diagonal edges than the vertical or horizontal edges. In the drawings, one diagonal grid space represents two horizontal or two vertical grid spaces.

4. a. 3 units by 4 units by 2 units

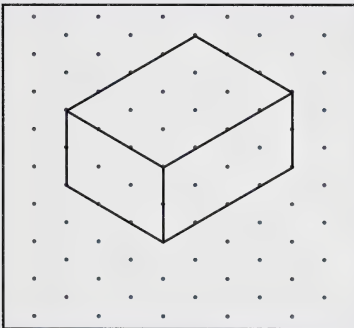


b. 5 units by 4 units by 3 units

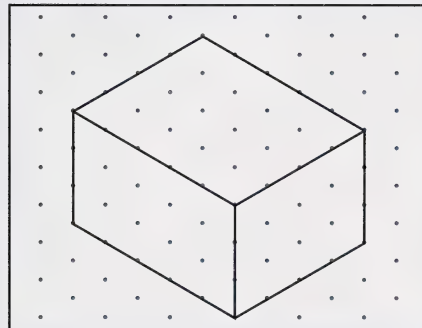


5. Using an isometric grid, all edges of the cube are the same number of grid units long. (**Note:** The number of grid units long is found by counting spaces, not dots.)
6. The angles formed by the edges of a cube drawn on an isometric grid are the same (each is 60°), but the angles drawn on an oblique grid may be different from one another.
7. The dots on the isometric dot paper represent the points where the lines cross on the isometric grid.

8. a. 3 units by 4 units by 2 units

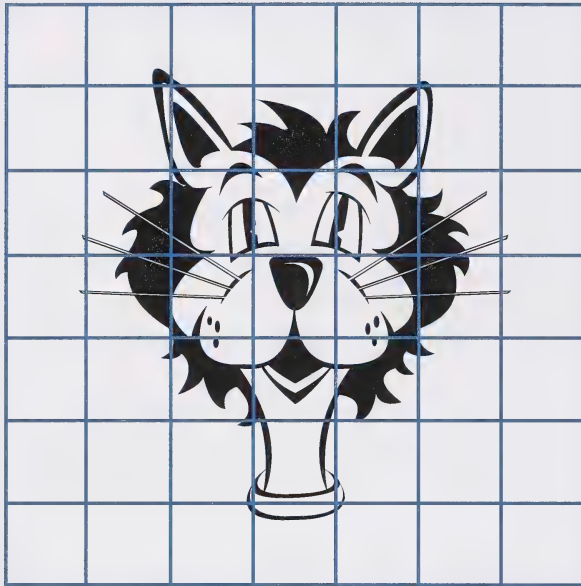


b. 5 units by 4 units by 3 units

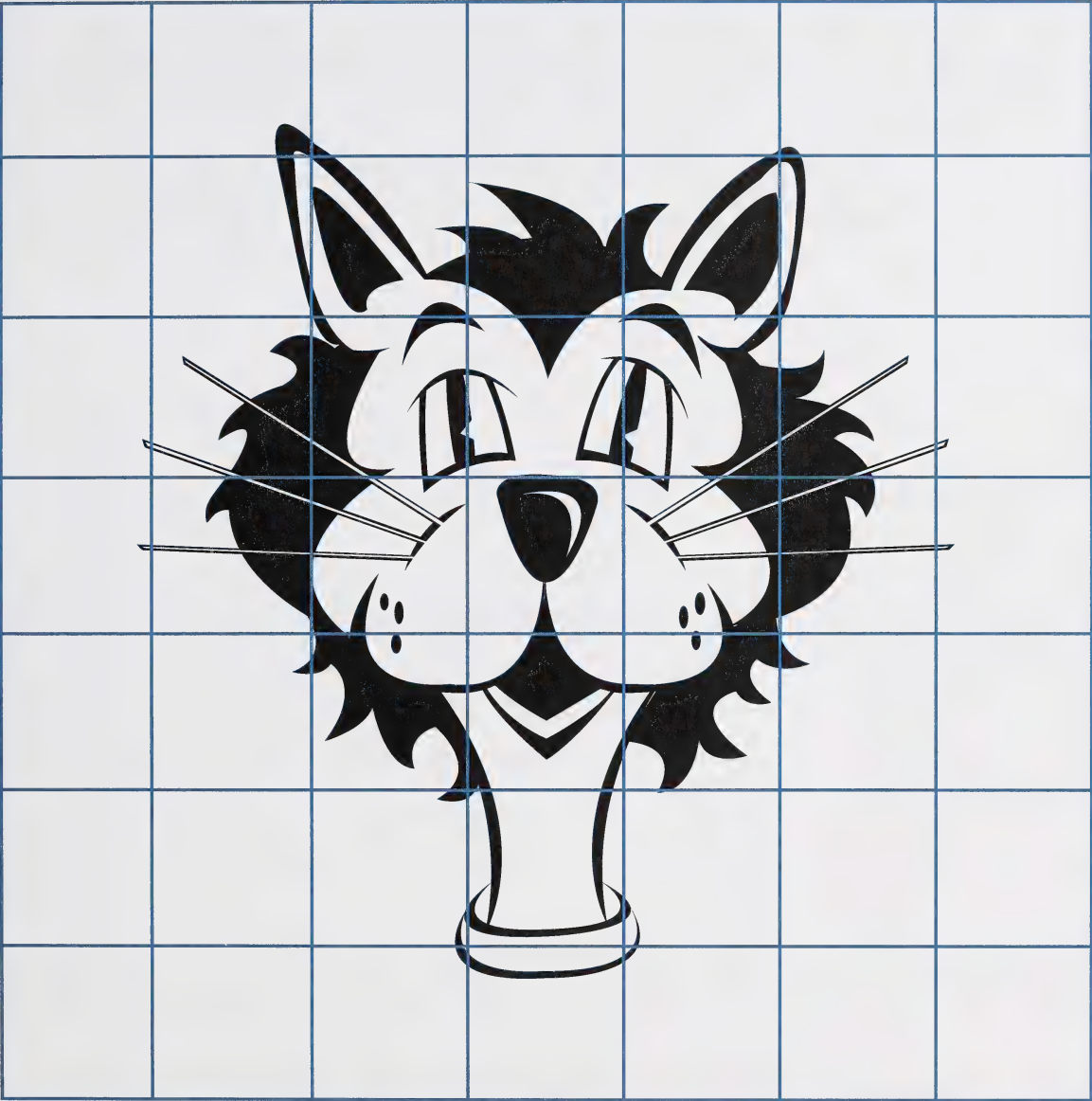


Activity 3

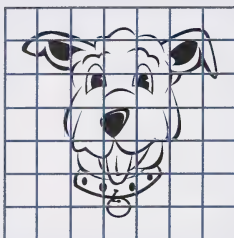
1.



2.



3.



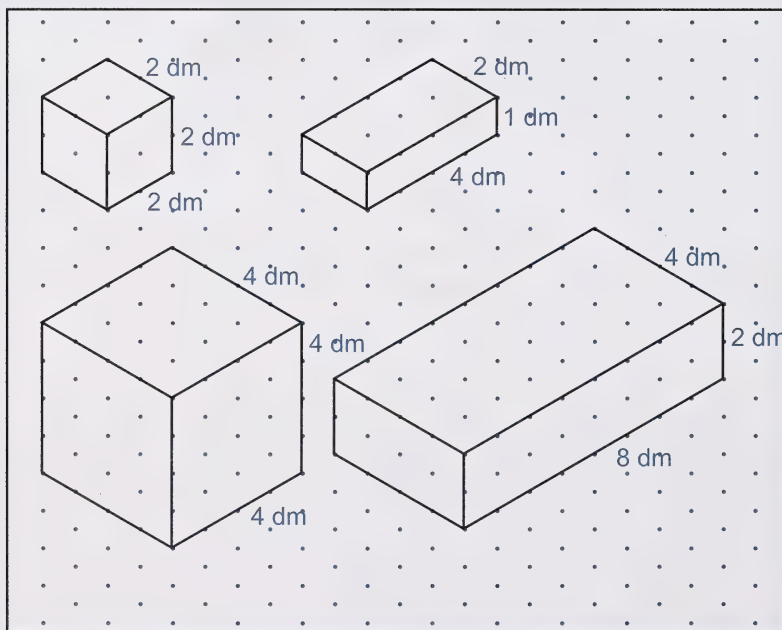
4. Answers will vary.

5. Answers will vary.

Challenge Activity

There are two possible solutions:

- The cube is $2\text{ dm} \times 2\text{ dm} \times 2\text{ dm}$ and the rectangular prism is $4\text{ dm} \times 2\text{ dm} \times 1\text{ dm}$.
- The cube is $4\text{ dm} \times 4\text{ dm} \times 4\text{ dm}$ and the rectangular prism is $8\text{ dm} \times 4\text{ dm} \times 2\text{ dm}$.

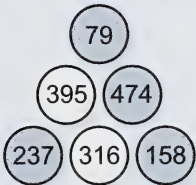


Keystrokes

1. a.

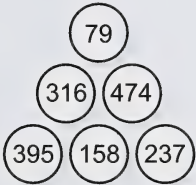
Keystrokes	ON/C	79	+	474	+	158	=	M+	–
Display	0	79	79	474	553	158	711	711	711

158	–	237	=	MR	–	237	–	79	=
158	553	237	316	711	711	237	474	79	395

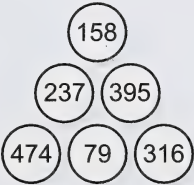


Magic sum = 711

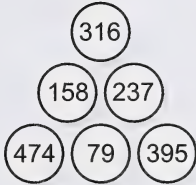
b. There are three other unique solutions.



Magic sum = 790



Magic sum = 869

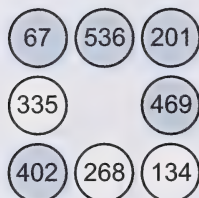


Magic sum = 948

2. a.

Keystrokes	ON/C	67	+	536	+	201	=	M+	–	201	–
Display	0	67	67	536	603	201	804	804	804	158	603

469	=	MR	–	134	–	402	=	MR	–	402	–	67	=
469	134	804	804	134	670	402	268	804	804	402	402	67	335



Magic sum = 804

b. There are five other unique solutions.



Magic sum = 871



Magic sum = 871



Magic sum = 938



Magic sum = 938



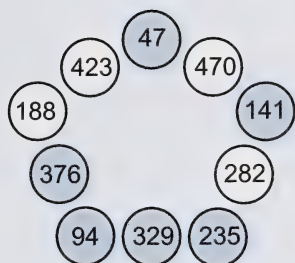
Magic sum = 1005

3. a.

Keystrokes	ON/C	94	+	329	+	235	=	M+	-	235
Display	0	94	94	329	423	235	658	658	658	235

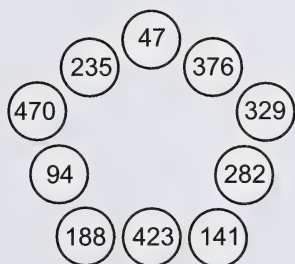
-	141	=	MR	-	141	-	47	=	MR	-	94
423	141	282	658	658	141	517	47	470	658	658	94

-	376	=	MR	-	188	-	47	=			
564	376	188	658	658	188	470	47	423			

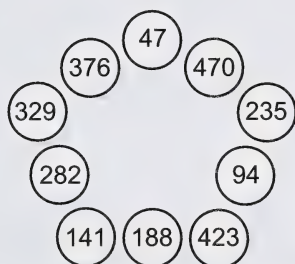


Magic sum = 658

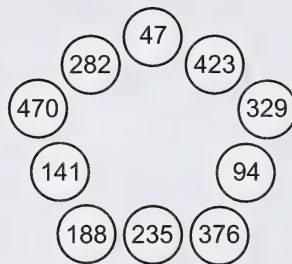
b. There are five other unique solutions.



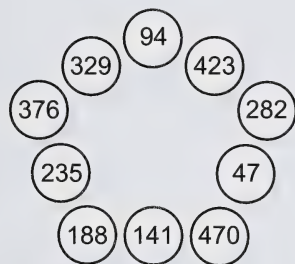
Magic sum = 752



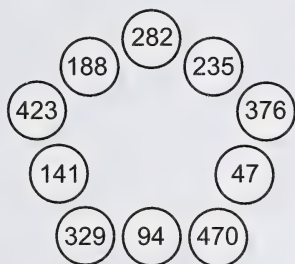
Magic sum = 752



Magic sum = 799



Magic sum = 799



Magic sum = 893

4. The number of cans in each layer is the layer number multiplied by itself.

5.

Keystrokes	ON/C	30	M+	5	×	=	M+	MR	6	×	=	M+	MR	7	×	=
Display	0	30	30	5	5	25	25	55	6	6	36	36	91	7	7	49

M+	MR	8	×	=	M+	MR	9	×	=	M+	MR	10	×	=	M+	MR
49	140	8	8	64	64	204	9	9	81	81	285	10	10	100	100	385

Layer Number	1	2	3	4	5	6	7	8	9	10
Number of Cans in the Layer	1	4	9	16	25	36	49	64	81	100
Total Number of Cans in the Pyramid	1	5	14	30	55	91	140	204	285	385

6. There are 1496 cans in a 16-layer pyramid.

Keystrokes	ON/C	385	M+	11	×	=	M+	12	×	=	M+	13	×
Display	0	385	385	11	11	121	121	12	12	144	144	13	13

=	M+	14	×	=	M+	15	×	=	M+	16	×	=	M+	MR
169	169	14	14	196	196	15	15	225	225	16	16	256	256	1496

7. Serby could make a pyramid with 20 layers if he had 3000 cans to stack. He would need 3311 cans to make 21 layers.

Keystrokes	ON/C	1496	M+	17	×	=	M+	MR	18	×	=	M+	MR
Display	0	1496	1496	17	17	289	289	1785	18	18	324	324	2109

19	×	=	M+	MR	20	×	=	M+	MR	21	×	=	M+	MR
19	19	361	361	2470	20	20	400	400	2870	21	21	441	441	3311

Review

1. Textbook, page 210, On Your Own, question 1

1. The yield sign has three acute angles.

The stop sign has eight obtuse angles.

The no-left-turn sign has four right angles, the arrow is a right angle, and the line through the circle is a straight angle.

The school sign has two right angles, two obtuse angles, and one acute angle. If you look closely, other angles appear. For example, the back of the boy's knee is an obtuse angle, and in front of his knee is a reflex angle.

2. Textbook, page 210, Practise Your Skills, questions 1 to 6

- | | | |
|----------------|-----------------|-----------------|
| 1. acute angle | 2. right angle | 3. obtuse angle |
| 4. acute angle | 5. obtuse angle | 6. acute angle |

3. a. Textbook, page 222, Skill Bank from This Unit, questions 1 and 2

- | | |
|-------------------------------|--------------------|
| 1. a. less than a right angle | 2. a. reflex angle |
| b. less than a right angle | b. straight angle |
| c. greater than a right angle | c. obtuse angle |

b. Textbook, page 247, Skill Bank Looking Back, question 4

4. a. acute angle b. straight angle c. obtuse angle

c. Textbook, page 271, Skill Bank Looking Back, question 1

1. a. right angle b. acute angle c. reflex angle

4. Textbook, page 211, Finding the Measures of Angles, questions 1 and 2

1. Each angle that touches the centre point is 60° . The measures of the other angles in each triangle are 60° . You can measure them with a protractor. All of the triangles are equilateral triangles.

2. • The yellow hexagon has six 120° angles.
 - The blue rhombus has two 120° angles and two 60° angles.
 - The blue rectangle has four 90° angles.
 - The orange square has four 90° angles.
 - The red trapezoid has two 60° and two 120° angles.

5. Textbook, page 212, Practise Your Skills, question 1 and 2

1. The angles labelled “a” are 90° .
2. The angles labelled “b” are 45° .

6. Textbook, page 215, Practise Your Skills, questions 1 to 3

1. The acute angle is about 26° .
2. The acute angle is about 46° .
3. The obtuse angle is about 134° .

7. a. Textbook, page 222, Skill Bank from This Unit, questions 3 and 4

3. a. Angles m , n , and p are 60° and angle r is 120° .
b. Angle d is 180° .
c. Angle h is 150° and angle k is 90° .

4. The angle is 40° .

b. Textbook, page 247, Skill Bank Looking Back, questions 5 and 6

5. a. Angle r is 90° and angle s is 180° .
b. Angle d is 120° and angle t is 60° .
6. a. The angle is 60° .
b. The angle is 120° .

c. Textbook, page 271, Skill Bank Looking Back, questions 2 and 3

2. The angles in an equilateral triangle are all 60° .
3. The measure of an angle that makes a square corner is 90° .

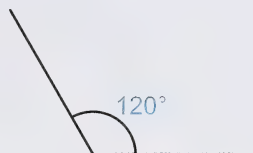
8. Textbook, page 215, Practise Your Skills, questions 4 to 9

4.



acute angle

5.



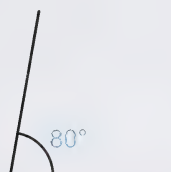
obtuse angle

6.



acute angle

7.



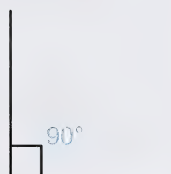
acute angle

8.



reflex angle

9.



right angle

9. a. Textbook, page 222, Skill Bank from This Unit, question 5

5. a.



b.



c.



d.



b. Textbook, page 271, Skill Bank Looking Back, question 4

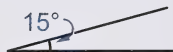
4. a.



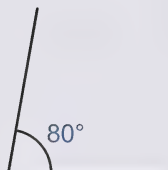
b.



c.

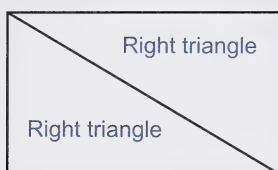


d.



10. Answers may vary. Sample answers are given.

a.



or

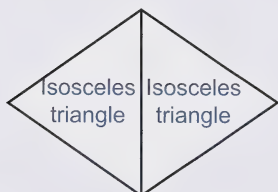


- quadrilateral, parallelogram, rectangle

- It is not a regular polygon.

- In the first example, it is not a line of symmetry, but in the second example, it is a line of symmetry

b.



- quadrilateral, parallelogram, rhombus

- It is not a regular polygon.

- It is a line of symmetry.

c.



or

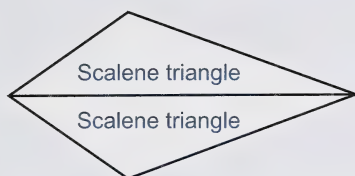


- hexagon

- It is a regular polygon.

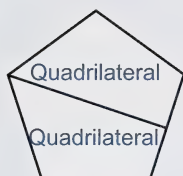
- In both examples, they are lines of symmetry.

d.



- kite
- It is not a regular polygon.
- It is a line of symmetry.

e.



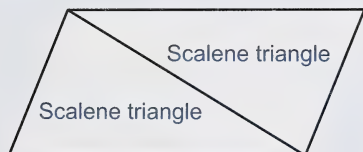
- pentagon
- It is a regular polygon.
- It is a line of symmetry.

11. a.



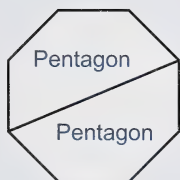
- quadrilateral, trapezoid
- It is not a regular polygon.
- It is a line of symmetry.

b.

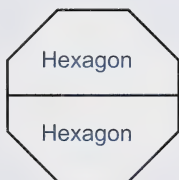


- quadrilateral, parallelogram
- It is not a regular polygon.
- It is not a line of symmetry.

c.



or



- octagon
- It is a regular polygon.
- In both examples, they are lines of symmetry.

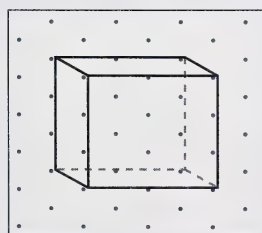
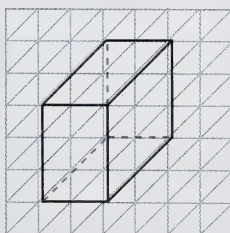
12. a. Textbook, page 232, Practise Your Skills, questions 1 to 6

1. square pyramid
4. triangular prism

2. pentagonal pyramid
5. rectangular prism

3. triangular pyramid
6. pentagonal prism

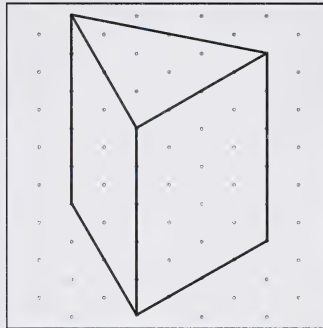
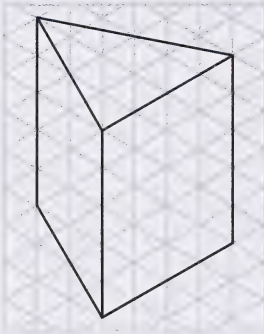
b.



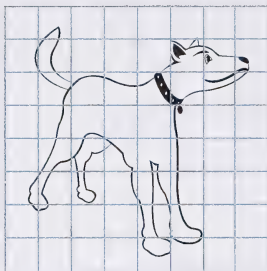
13. a. Textbook, page 236, Practise Your Skills, questions 1 to 4

1. pentagonal prism: 7 faces, 10 vertices, and 15 edges
2. triangular prism: 5 faces, 6 vertices, and 9 edges
3. rhombohedron: 6 faces, 8 vertices, and 12 edges
4. octagonal pyramid: 9 faces, 9 vertices, and 16 edges

b.



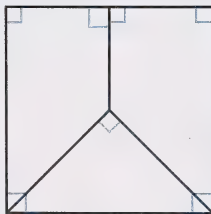
14.



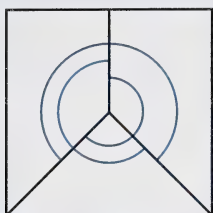
15. Answers will vary.

16. Textbook, pages 220 to 221, Problem Bank, questions 1, 2, 3, 4, and 6

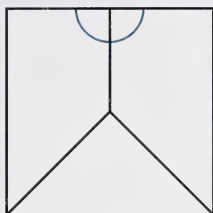
1. There are seven right angles.



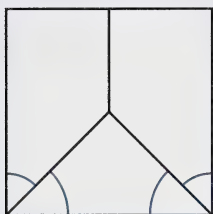
2. a. three reflex angles



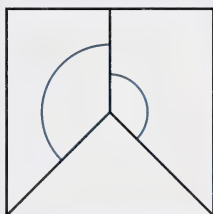
b. one straight angle



c. four acute angles

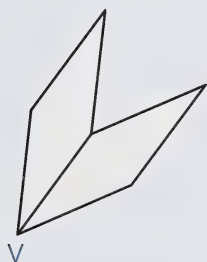


d. two obtuse angles



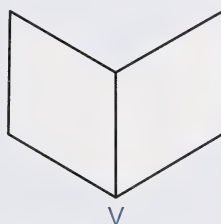
3. Answers will vary. Sample answers are given. (**Note:** The vertex of each angle being referred to is labelled V.)

a. an acute angle



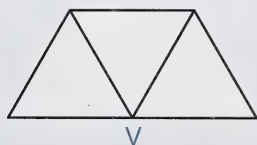
$$30^\circ + 30^\circ = 60^\circ$$

b. a 120° angle



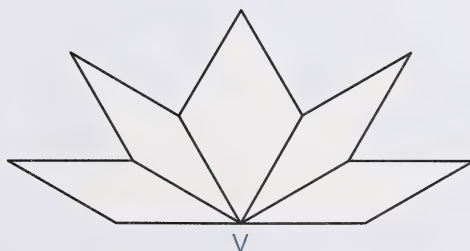
$$60^\circ + 60^\circ = 120^\circ$$

c. a straight angle



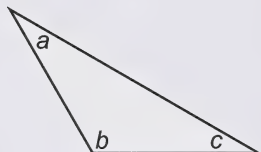
$$60^\circ + 60^\circ + 60^\circ = 120^\circ$$

d. a 180° angle



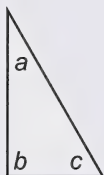
$$30^\circ + 30^\circ + 60^\circ + 30^\circ + 30^\circ = 180^\circ$$

4. a. The acute angle inside the tan rhombus is 30° .
 b. The obtuse angle inside the tan rhombus is 150° .
6. a. Yes, if a triangle has an obtuse angle, the other two angles must both be acute.



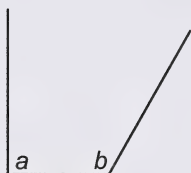
Angle b is obtuse, so angles a and c must be acute. Otherwise, the figure would not close to form a triangle.

- b. Yes, a triangle can have a 90° angle, a 30° angle, and a 60° angle.



Angle b is 90° , angle a is 30° , and angle c is 60° .

- c. No, a triangle cannot have an obtuse angle, a right angle, and an acute angle.



Angle a is a right angle (90°) and angle b is obtuse. The figure does not close to form a triangle.

- d. No, a triangle cannot have two right angles and one acute angle.



Angle a and angle b are both right angles (90°). The figure does not close to form a triangle.

- e. A triangle cannot have a 180° angle, a 30° angle, and a 90° angle.

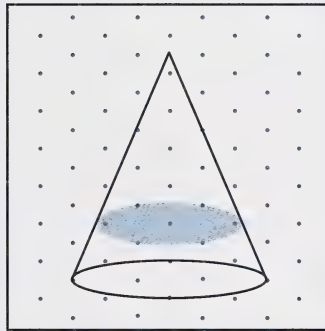
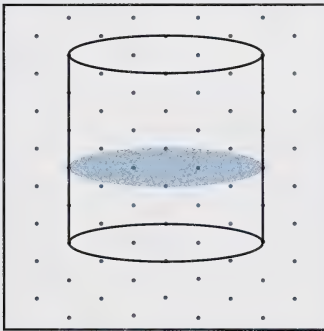
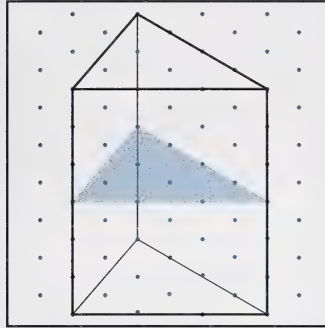
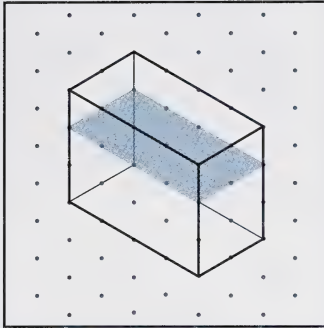


180° is a straight line. The figure cannot close to form a triangle.

17. Textbook, pages 244, Problem Bank, question 2

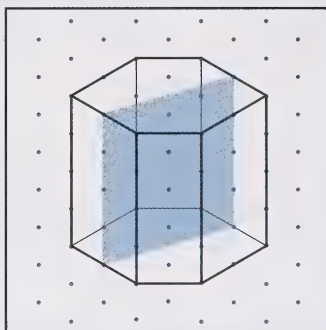
2. a. The slice is a hexagon because it is the same shape as the bases.

b.



c. For cylinders and prisms whose vertical edges are parallel to each other, the shape or size of the horizontal cross section does not depend upon how high or low the slice is made. It will always be the same size and shape as the bases. For cones, the shape or size of the horizontal cross section will always be a circle. However, the size of the circle depends upon how high or low the slice is made.

- d. The vertical cross sections of the hexagonal prism will be rectangles, as shown by the one at the right:



Just the Facts

Multiplication and Division Facts

5	6	6	8	48
5	14	1	4	7
0	2	45	4	21
9	0	0	36	3
64	5	48	2	0

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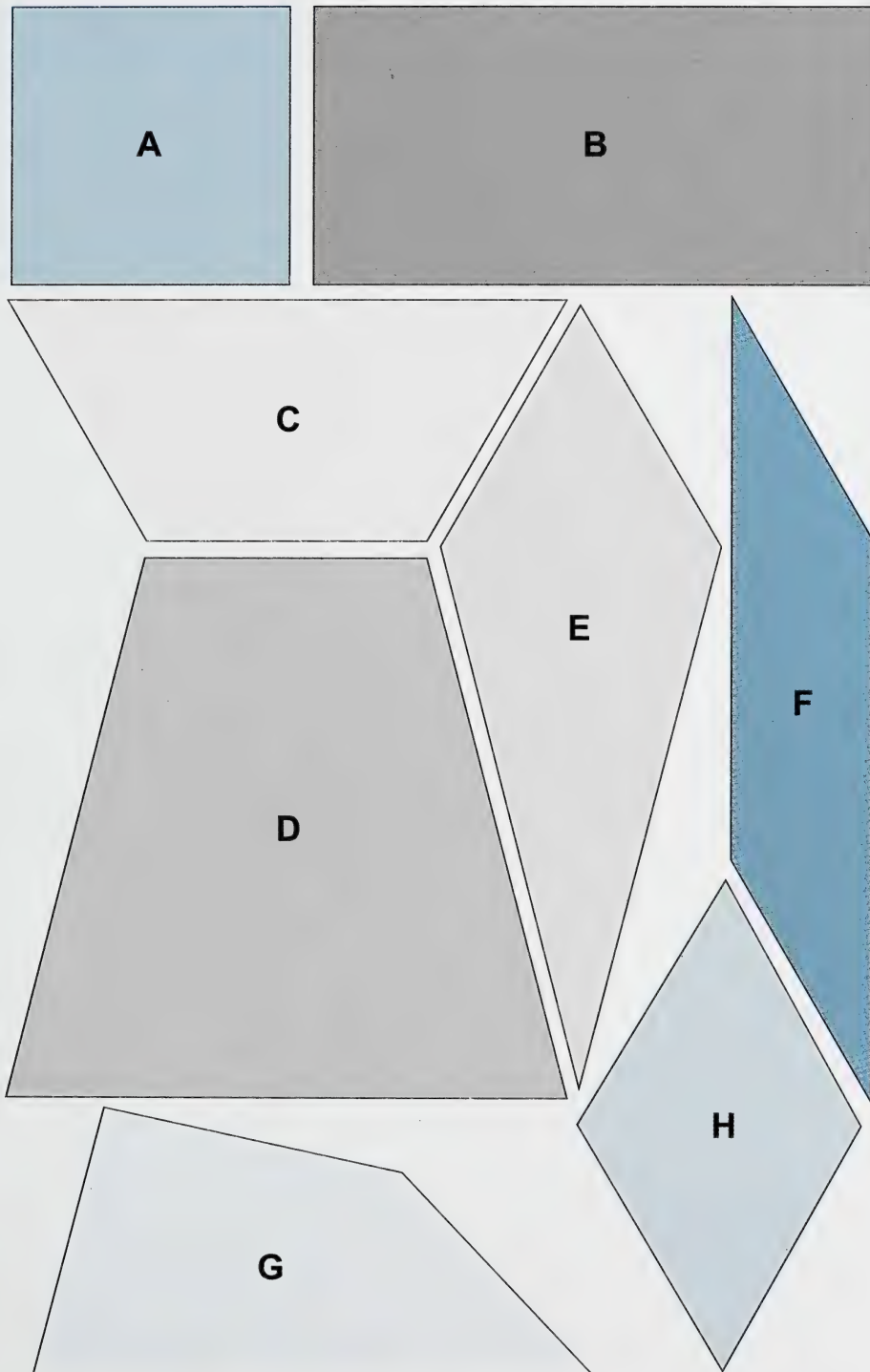
Introductory pages: NASA/ASC (all)

Page

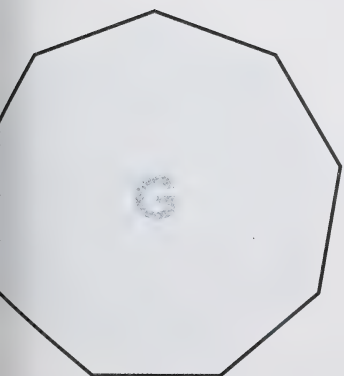
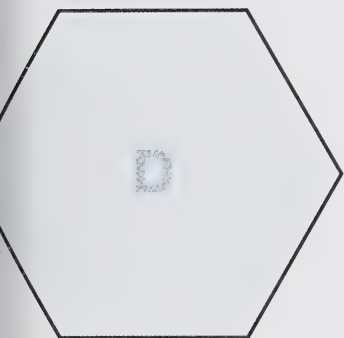
8	EyeWire Collection/Getty Images
10	PhotoDisc Collection/Getty Images (bottom)
12	Corbis
21	PhotoDisc Collection/Getty Images (both)
46	PhotoDisc Collection/Getty Images
47	© 2002 www.clipart.com (collage)
51	PhotoDisc Collection/Getty Images
57	PhotoDisc Collection/Getty Images
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Learning Aids

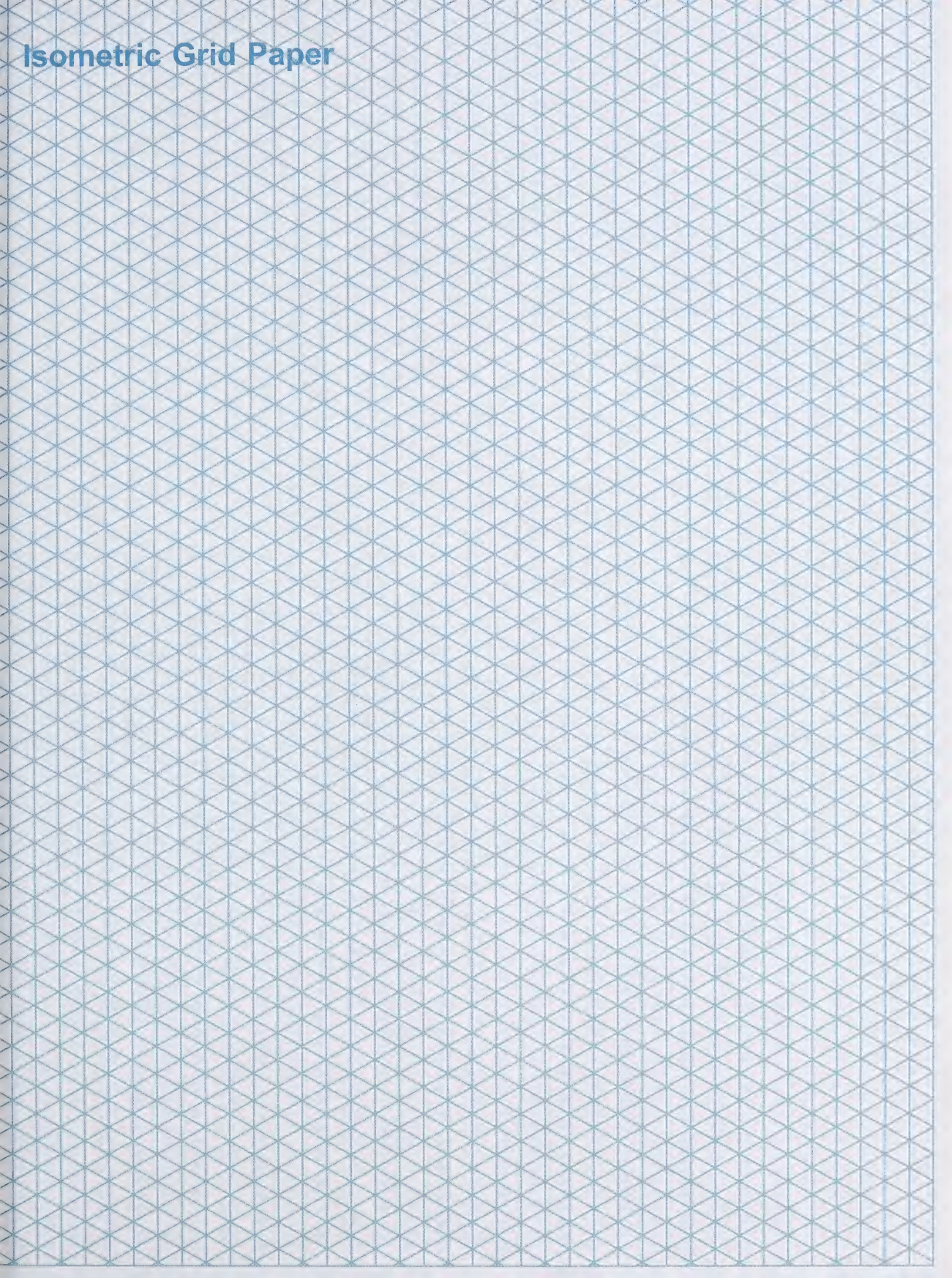
Pattern Shapes

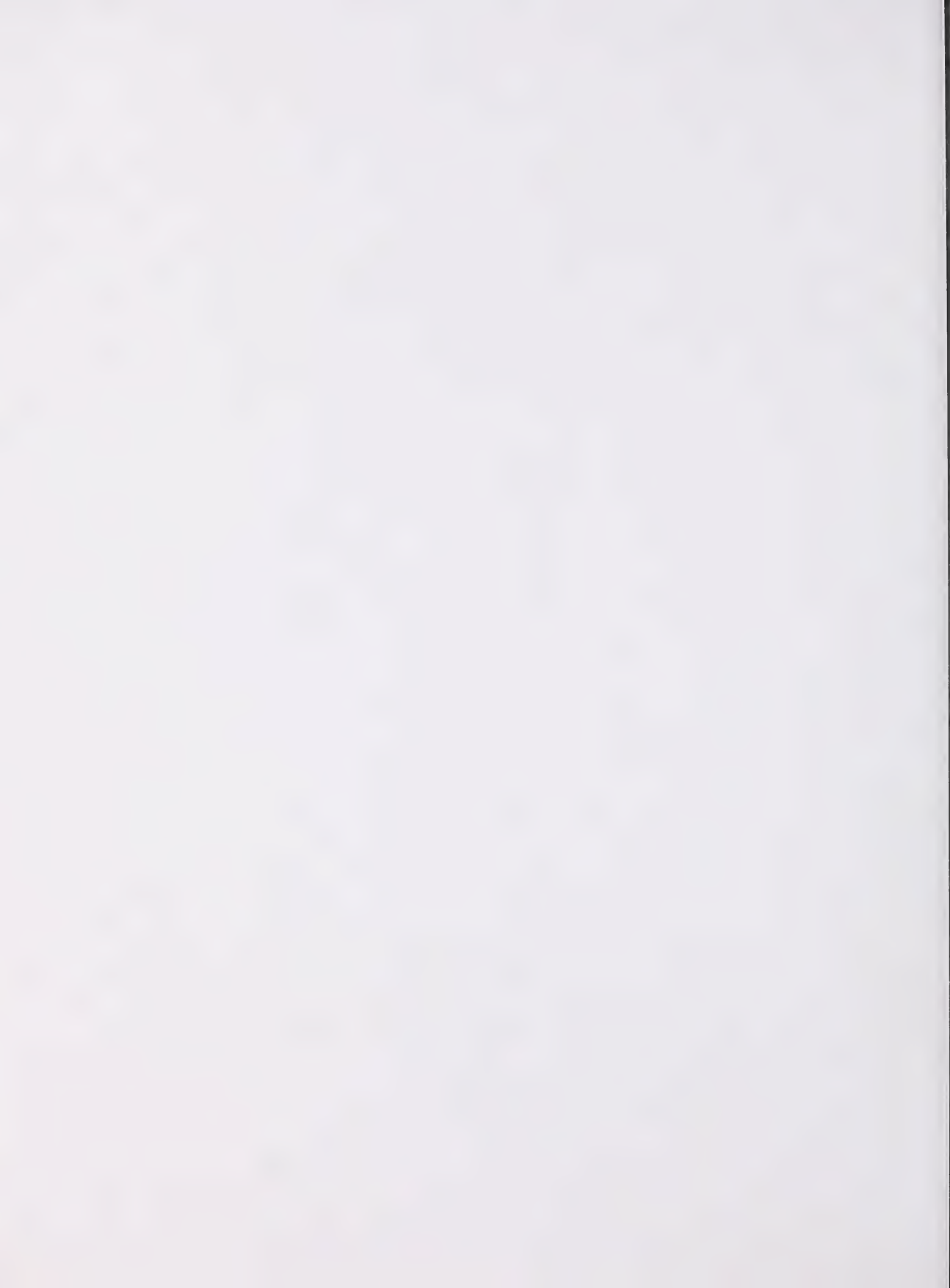


Regular Polygons



Isometric Grid Paper

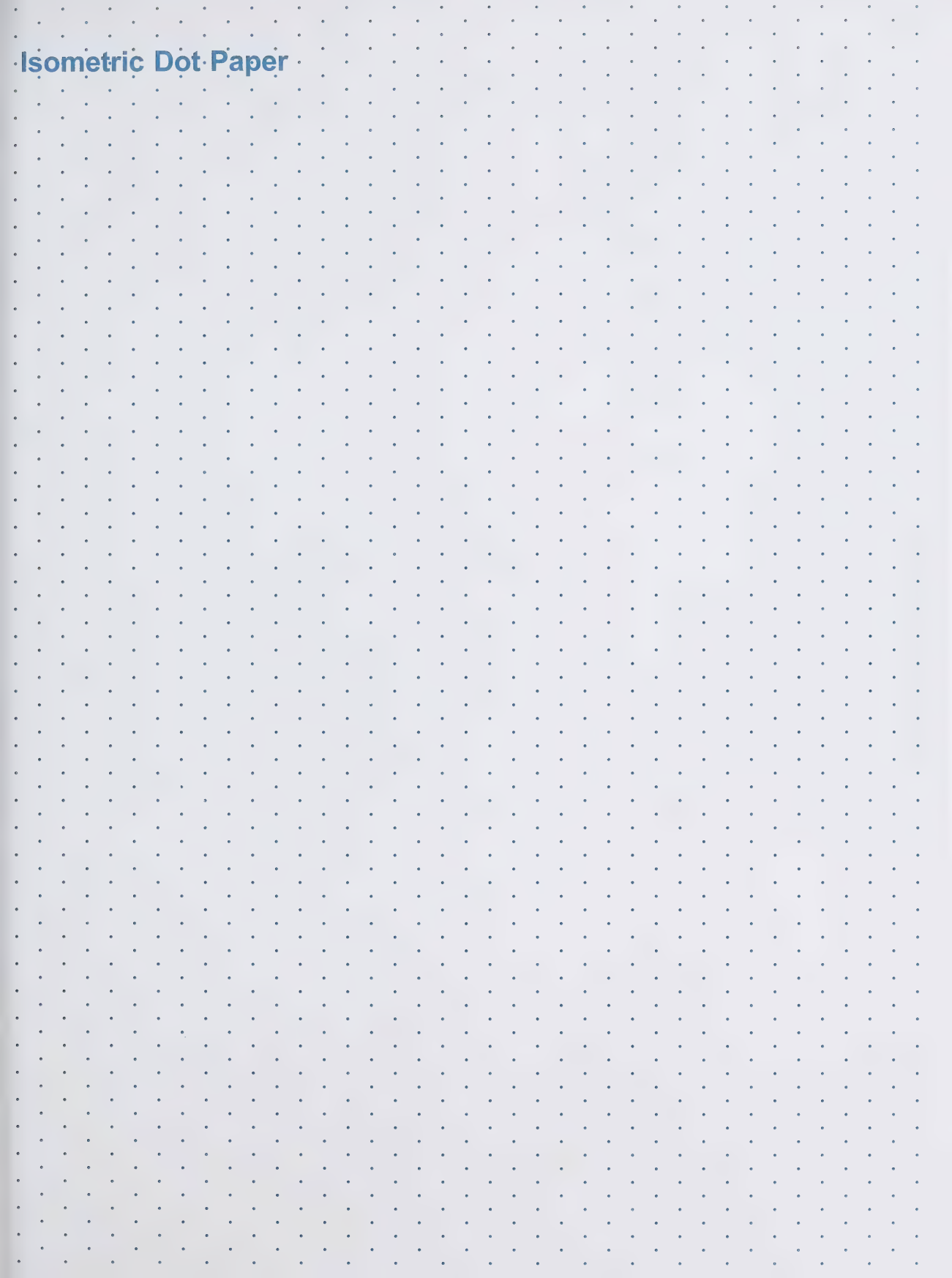




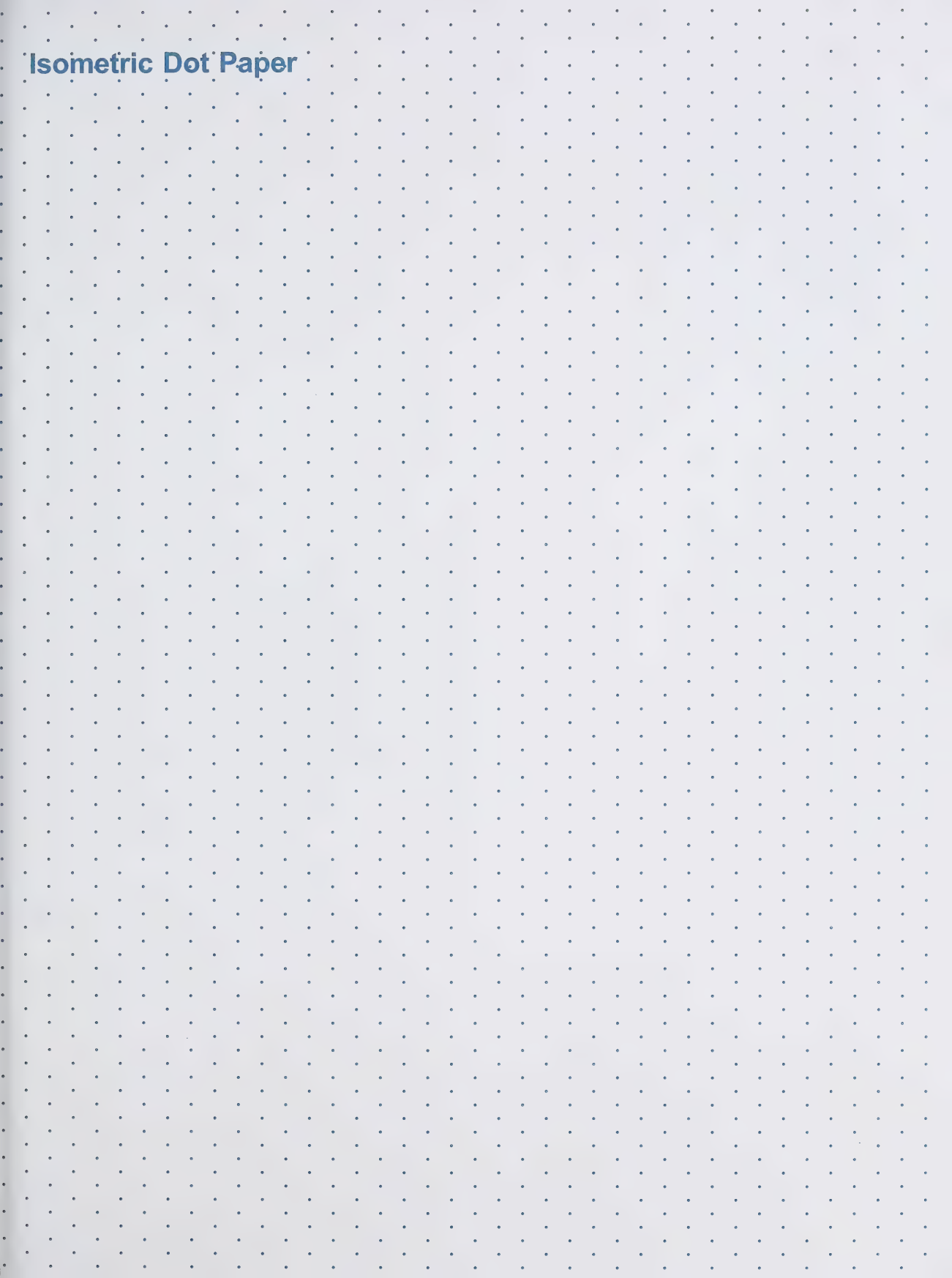
Oblique Grid Paper



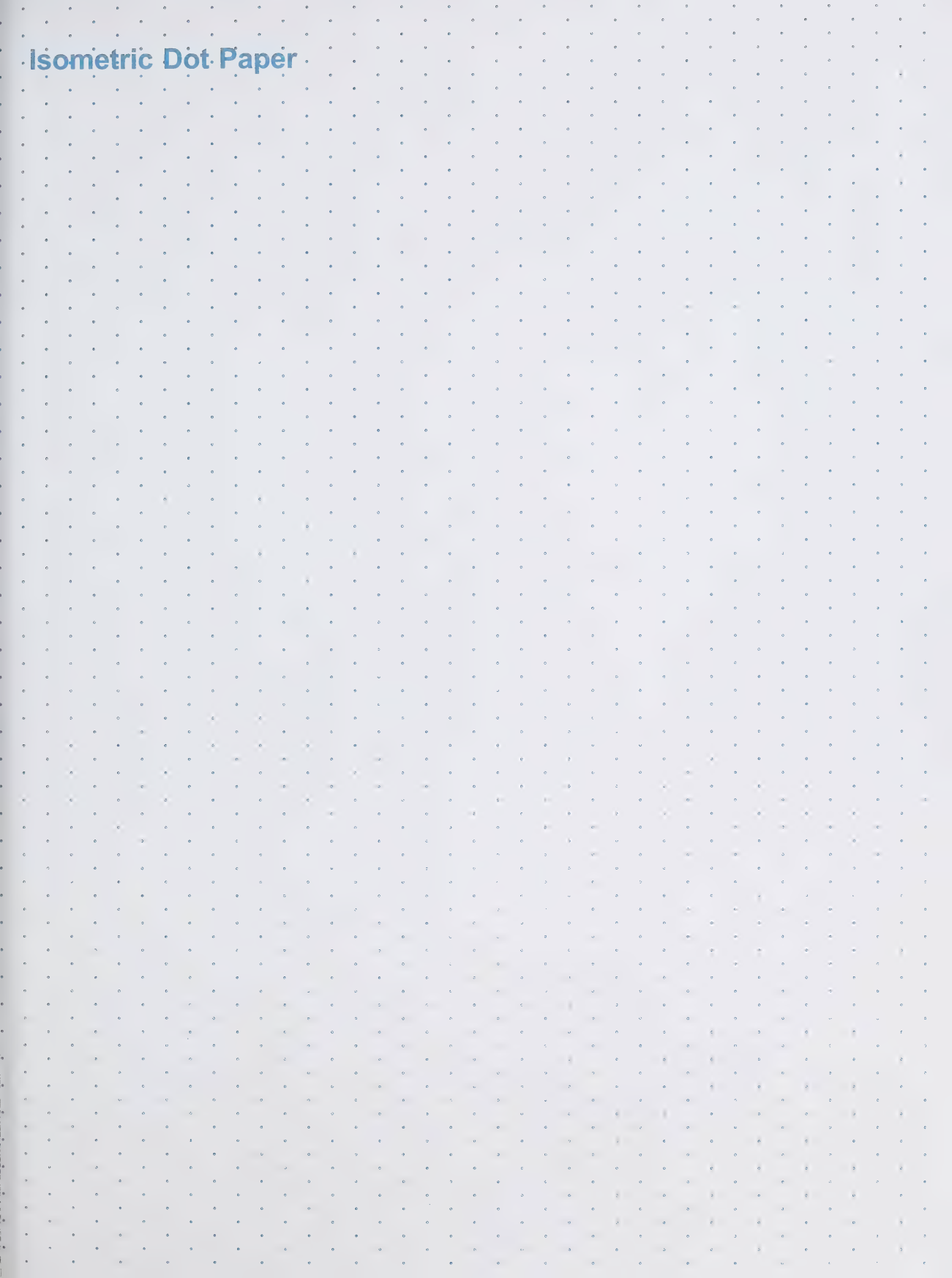
Isometric Dot Paper



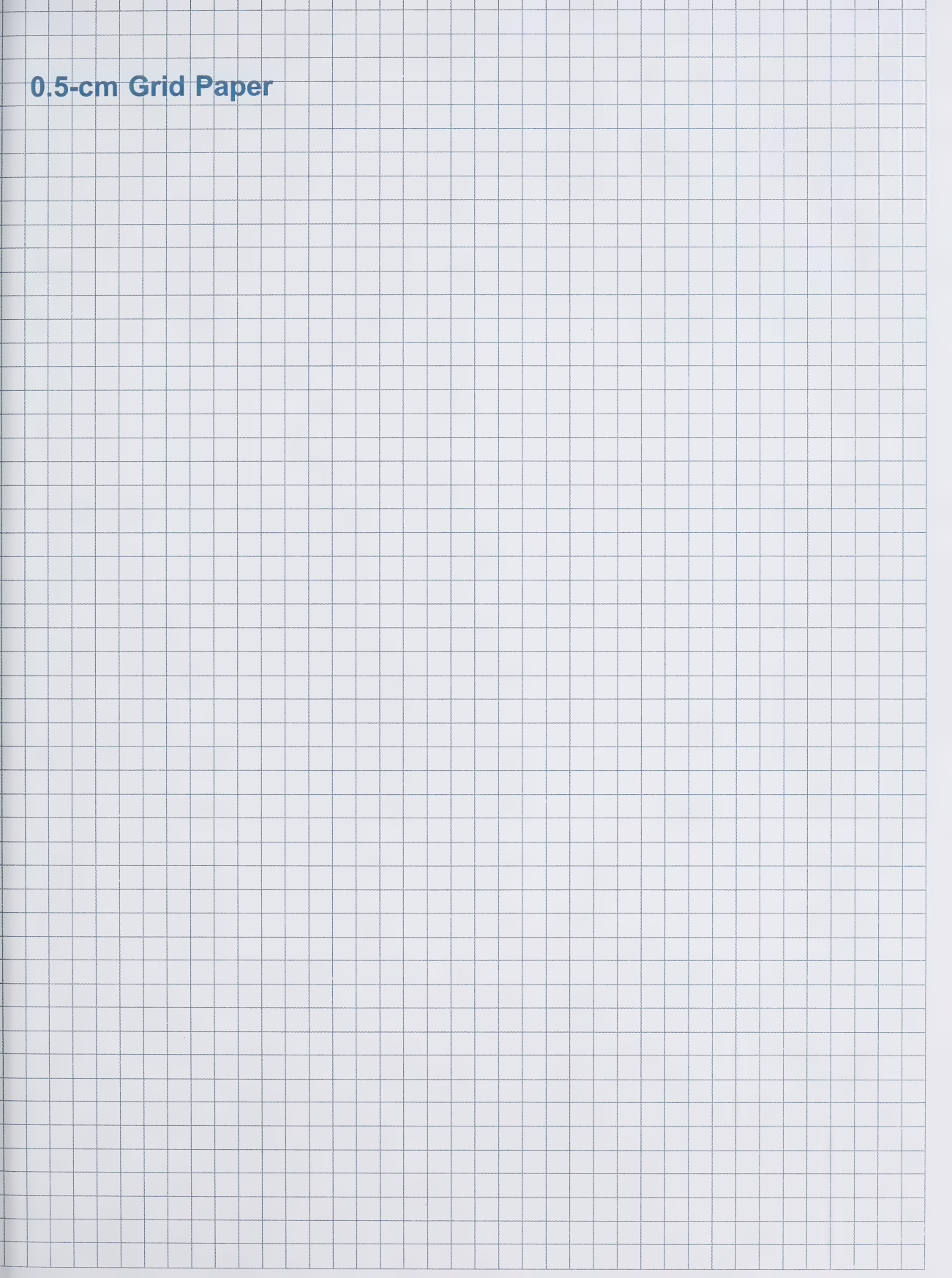
Isometric Dot Paper



Isometric Dot Paper



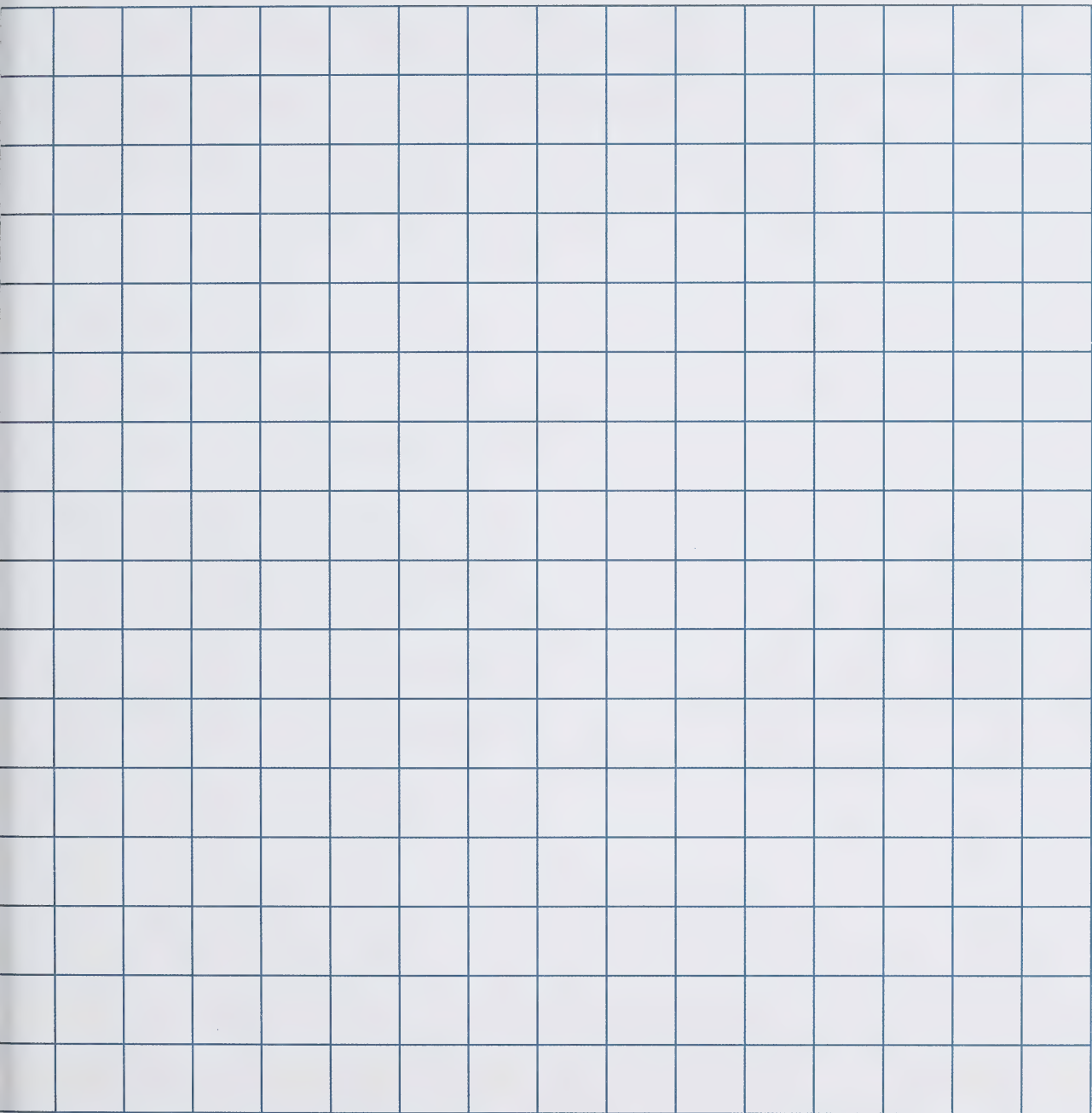
0.5-cm Grid Paper



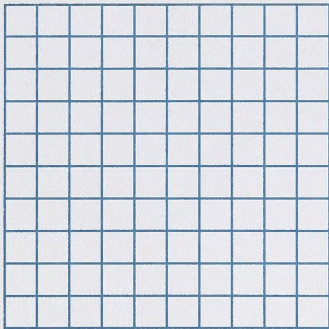
2.5-cm Grid Paper

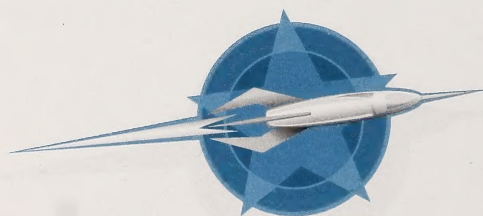


1.25-cm Grid Paper



0.5-cm Grid Paper





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